

PEIRCE POLYMATH

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The term ‘polymath’ is referred to a person of wide knowledge, as exemplified by Renaissance’s sages. Open to the World, the polymath exerts an action both large and profound over the subjects under study. *Depth and breadth* go hand by hand, in an exploration attentive to incisive technical advances, but also to surrounding universal visions, beyond mere specialized practices. In his studies on the history of science¹, Peirce exhibited long lists of polymaths, a scientific tradition which he pretended to emulate. His architecture of knowledge – around mathematics, logic, philosophy, semiotics, science studies, classification methods – covers a gigantic range, both on very ample, general, perspectives, and on extremely precise, particular, case studies. Here we will focus on Peirce’s «polymath mathematics», that is, on his multiverse understanding of mathematics. It is a small section of his immense philosophical design, but rich enough to offer a *full and faithful*² glimpse of the entire architectonics.

In the next pages, we navigate briefly through some of Peirce’s main outputs in mathematics, trying both to present their (differential) specificity and their (integral) connections. *Section 1* describes his main standard contributions in mathematical logic (propositional calculus, relatives, quantifiers, the logic of number). *Section 2* delves into his nonstandard logical inputs (existential graphs, Peircean continuum). *Section 3* focuses on searches in combinatorics and topology (relational graphs, Listing numbers, four colors theorem). *Section 4* offers a quick stroll through Peirce’s applied mathematics (geodesic, statistics, computing, cartography, aerodynamics). *Section 5* elaborates an outlook of how his general philosophical system («polymath») is pervaded by mathematical orientations («polymath mathematics»).

1. Standard Mathematical Logic

Peirce’s two main contributions to the *propositional calculus* are *On the Algebra of Logic* (1880)³ and *On the Algebra of Logic. A Contribution to the Philosophy of Notation* (1885)⁴. Peirce situates himself in the logical tradition of Boole and De Morgan, but the

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¹ Cfr. C.S. Peirce, *Historical Perspectives on Peirce’s Logic of Science. A History of Science*, edited by C. Eisele, Parts I/II, Mouton de Gruyter, Berlin 1985.

² In terms of the mathematical Theory of Categories, a functor is *full* if it captures the (full) *integrality* of what is to be represented, and it is *faithful* if it captures the (faith) *differentiability* of the realm under scrutiny. An *integral and differential* calculus lies at the bottom of any universal (*unum-versus-alia*) approach to knowledge. Cfr. P. Florenskij, *La colonna e il fondamento della verità*, Mimesis, Milano 2012.

³ C.S. Peirce, *On the Algebra of Logic*, «American Journal of Mathematics», 3, 1880, pp. 15-57.

⁴ Id., *On the Algebra of Logic. A Contribution to the Philosophy of Notation*, «American Journal of Mathematics», 7, 1885, pp. 180-196.

young logician goes well beyond his predecessors, and studies foundations, definitions, axioms, before proceeding to the development of the calculus. In the first article, Peirce introduces symbols for proof and implication, and distinguishes forms of *modus ponens* and the *deduction theorem*⁵. Then, Peirce defines what will be understood as the *intuitionistic negation* $\neg p \leftrightarrow (p \rightarrow \perp)$ and presents some basic principles of the algebra of implication (identity, contradiction, excluded third). Here Peirce presents what will be later called «Peirce's law» $((p \rightarrow q) \rightarrow p) \rightarrow p$, and proves that the excluded third may be deduced from this law⁶. The second article pursues further Peirce's searches around foundations, and offers a *complete axiomatic* presentation of the classical propositional calculus. Further, not far away from Post and Wittgenstein's later treatment of truth tables (1920), Peirce offers a procedure to prove how a general implicative formula would be a tautology: through recursive processes, he tries to falsify it, and at the end he obtains contradictions.

The two articles on the algebra of logic (1880, 1885) are part of a fabric – together with *Description of a Notation for the Logic of Relatives* (1870)⁷, and *On the Logic of Relatives* (1883)⁸ – where Peirce exposes his results related to *first order logic*. In the 1870 article, Peirce offers varied developments of the algebra of relations, with emphasis on the mathematical methods (operations, differentiation) by Boole and De Morgan. In the third part of the article of 1880, we wander through atoms, operations, relations, and limits. In the 1883 article, binary relative multiplication and sum are defined through infinitary products and summations. Finally, in the 1885 article, Peirce uses Σ and Π for our actual \exists and \forall , and shows the utility of the new quantifiers for processes of symbolization, and for an algorithmic construction of normal forms. After fifteen years (1870-1885) dedicated to the algebra of logic, Peirce possesses thus a very elastic representational web («polymath logical calculi»: symbols, connectives, axioms, correlations, quantifiers, limits) which captures some of the main problems in logic.

On the other hand, Peirce's contribution to a *structural* understanding of the natural numbers is very interesting. *On the Logic of Number* (1881)⁹ proposes distinctions between discrete and continuous systems, and offers axioms (in natural language) for a minimum, provides recursive definitions for sum and product, and proves associativity, commutativity, distributivity¹⁰. Several forms of induction principles are proved in their adequate context. Once the natural numbers have been

⁵ Compare this with Frege's very complicated proofs in his *Begriffsschrift*, where six pages devoted to proving paragraph 16 could be reduced to few simple lines, if Frege had a deduction theorem. Cfr. J. van Heijenoort, *A Source Book in Mathematical Logic*, Harvard University Press, Harvard 1967, pp. 37-43.

⁶ In fact, Peirce is here anticipating intuitionism in several ways, and is *proving* beforehand that the classical and intuitionistic propositional calculi *differ exactly* around Peirce's law. Many years, and the emergence of the Polish logical school, would be needed before this could be entirely understood.

⁷ C.S. Peirce, *Description of a Notation for the Logic of Relatives, Resulting from an Amplification of the Conceptions of Boole's Calculus of Logic*, «Memoirs of the American Academy of Arts and Sciences», 9, 1873, pp. 317-378.

⁸ Id., *A Theory of Probable Inference. Note B. The Logic of Relatives*, in *Studies in Logic by Members of the Johns Hopkins University*, John Hopkins, Boston 1883, pp. 187-203.

⁹ Id., *On the Logic of Number*, «American Journal of Mathematics», 4, 1881, pp. 85-95.

¹⁰ Connected to Bolzano, Grassmann, Dedekind, or Peano, Peirce's axiomatic, definitional, recursive approach to the natural numbers has not been appreciated enough.

constructed, the integers appear as inverting the discrete infinite through both directions. Further, at the end of his 1885 algebra of logic article¹¹, Peirce offers *two definitions* of *finite* set, one using second order quantifiers (formalizing Dedekind's idea, a set is finite if it cannot be put in bijective correspondence with a strict part), and the other one using first order quantifiers (expressing that a set is finite if every 1-1 endofunction on it must also be onto it). With all of this in perspective, Peirce should already be considered as an exceptional «polymath in mathematical foundations» – but we will see that this is only a small part of a truly universal vision, understood through his later mathematical standpoints.

2. Nonstandard Mathematical Logic

Arguably, Peirce's logic *depth and breadth* acquire its major features around his *existential graphs* (EG) and the *continuum*. Passing from the discrete to the continuous, from algebra to topology, mathematical logic enters *multidimensional* realms which forcefully extend problems, perspectives, methods, techniques, related to both the *foundations* and the *practice* of mathematics. EG, considered by Peirce himself as his *chef d'oeuvre*¹², cover diagrammatically a wide diversity of logics, ranging from classical propositional calculus (Alpha graphs), to classical first-order logic (Beta graphs), to modal calculi (Gamma graphs I) and second-order logic (Gamma graphs II)¹³. Here we are in presence of the unifying vision of a «polymath», embodied in an amazing array of multiple technical achievements. *Breadth* is obtained through the simultaneous axiomatization of a *diversity* of propositional calculi¹⁴ and purely relational first-order logic. *Depth* is achieved through *five uniform generic rules* (double alpha cuts, insertion, erasure, iteration, and deiteration), which offer explicit technical *common* roots for logical calculi. The same rules detect, in the context of Alpha language, the handling of classical negation and conjunction and, in the context of Beta language, the handling of the existential quantifier: something just unimaginable for any logic student raised into Hilbert-type logic systems. The EG calculi show thus that there exists a *kernel*, a «real general» for logical thought, which in some representational contexts gives rise to the propositional modes of connection, and in other contexts gives rise to the classical modes of quantification. A «polymath» *continuity* appears between discrete Alpha propositional forms and continuous Beta existential forms, offering a universal archetypal comprehension of particular logical types.

The EG's continuity characteristics (involved in *topological movements* around

¹¹ If one were to summarize all that Peirce said in this fifteen-page text, it must surely be regarded as one of *the greatest gems in the history of logic*.

¹² Letter to Jourdain, 1908. Cfr. Don Roberts, *The Existential Graphs of Charles S. Peirce*, Mouton, The Hague 1973, p. 110.

¹³ In the last twenty years of his life, Peirce devoted many manuscripts to the EG. The most complete published presentation appears in his *Lowell Lectures* (1903). Cfr. Charles S. Peirce, *The 1903 Lowell Lectures*, edited by A.-V. Pietarinen, De Gruyter, Berlin 2021 (*Peirceana Volume 2,2*).

¹⁴ Arnold Oostra has amplified the range of EG even to *intuitionistic* calculi (2010) and *geometrical* extensions over the sphere, the cylinder, the torus (2019). Cfr. A. Oostra, *Intuitionistic and Geometrical Extensions of Peirce's Existential Graphs*, in *Advances in Peircean Mathematics. The Colombian School*, edited by F. Zalamea, De Gruyter, Berlin 2023, pp. 105-180.

the sheet of assertion and its extensions to general complex variable settings¹⁵) are part of a more general picture, related to Peirce's *continuum*. At the center of his philosophical, methodological, and semiotic edifice, the continuum pervades completely Peirce's architecture. Far from being «a castle in the air»¹⁶, Peirce had very precise views on how to handle the continuum¹⁷. Going well beyond Cantor's real numbers system, accepting infinitesimals, introducing strong *reflexive* and *supermultitudinousness* properties, and delving into *modalities*, Peirce opened a vast field of inquiry to probe a generic continuum, of which Cantor's construction was to become just «a first embryo of continuity»¹⁸. Beyond Peirce's many visions and clues for the continuum, a *full global* model which encompassed all of its generic/supermultitudinous, reflexive/inextensible, and modal/plastic characteristics was still to be imagined, against all cutting prejudices of the «experts» involved. As often happens in mathematics, the solution turned out to be as simple as it was profound. A century later, Francisco Vargas (2015)¹⁹ combined two powerful ideas to produce a straightforward ZFC model for Peirce's continuum, something that seemed in principle very difficult, or almost impossible: (1) first, a sheaf of copies of the real line is *iterated* along the class of all ordinals; (2) second, the lexicographic order relation obtained in the iterated model is *inverted* («Order *E*»). The result produces an infinitely ordinal-iterated tree of real lines, with its branches looking down (via *E*). Through that extremely simple feature, any local, partial cut in the tree («Monad») turns out to be isomorphic to the global, whole tree (six lines in the proof!). From there, the main generic/supermultitudinous, reflexive/inextensible properties of Peirce's continuum are obtained at once, and, with some more detail, modalities can be defined and developed through ordinal levels and ramifications. There are no more points whatsoever, only extended parts, and a continuous weldedness governs the model, as predicted by Peirce.

Combining the existential graphs and the continuum – of which the first may be considered a *reflection*²⁰ of the second –, we obtain an unexpected *nonstandard fabric*, solid and elastic at the same time, which helps to understand the wide and

¹⁵ Cfr. F. Zalamea, *Towards a Complex Variable Interpretation of Peirce's Existential Graphs*, in *Ideas in Action. Proceedings of the Applying Peirce Conference*, edited by M. Bergman, S. Paavola, A.-V. Pietarinen, H. Rydenfelt, Nordic Pragmatism Network, Helsinki 2010, pp. 254-264; A. Hugueth, *Topos of Existential Graphs over Riemann Surfaces*, Undergraduate Thesis, Universidad Nacional de Colombia, Bogotá 2022.

¹⁶ M.G. Murphey, *The Development of Peirce's Philosophy*, Harvard University Press, New York 1961, p. 407.

¹⁷ Peirce's writings on the continuum are fairly scattered in his manuscripts. The most complete presentation appears in Charles S. Peirce, *The New Elements of Mathematics*, edited by C. Eisele (4 vols.), Mouton, The Hague 1976. The continuum is particularly addressed in volume 3.

¹⁸ For a complete description of Peirce's continuum, see F. Zalamea, *Peirce's Logic of Continuity*, Docent Press, Boston 2012 (extended translation of *El continuo peirceano*, Universidad Nacional de Colombia, Bogotá 2001).

¹⁹ F. Vargas, *A Full Model for Peirce's Continuum*, in *Advances in Peircean Mathematics. The Colombian School*, edited by F. Zalamea, De Gruyter, Berlin 2023, pp. 55-103.

²⁰ The EG lives in the complex plane, part of Peirce's continuum, and some of its characteristics help to provide a foundation for the continuum (see below, *Section 5*, proof of the pragmatist maxim). But, in the more general terms of Mathematical Category Theory, one can imagine an *immersion* of the EG in the continuum, for which an associated *adjunction* would offer a precise reflection.

profound scope of mathematical logic. A «double polymath» approach, both at the level of the EG (*universal* kernel of particularities) and at the level of the continuum (*global* patch of localities), governs what may seem a «totally disconnected» panorama²¹. Apparently diverse logical strata (connectives, modalities, quantifiers, axioms, rules, cutting, gluing, etc.) become welded together under extended and deep geometrical adjunctions, something that a «polymath» cherishes at heart.

3. Combinatorics and Topology

Section 1 described briefly Peirce's algebraic logic, and *Section 2* did so with Peirce's topological logic. But, independently of logic, Peirce was very keen to explore the pendulum between the discrete and the continuous, around *combinatorics* on the one hand, and *geometry* on the other. Peirce's initial interest in combinatorics stemmed from his chemical studies²², and from the study of his father's associative algebras. Going beyond the chemistry environment²³ and the mathematical algebraic environment, Peirce quickly became interested in semiotics at large, where fairly general *combinations of signs* govern the field²⁴. A beautiful, little-known diagram by the 20-years-old Peirce encodes this drive for classifications and combinations (see *Figure 1*, «Diagram of the It»).

²¹ This recalls *Stone's representation theorem*, in which Cantor's triadic set governs universally *all* compact Hausdorff totally disconnected spaces.

²² Harvard bachelor of science degree in chemistry, 1863, *summa cum laude*.

²³ Chemistry *valences* will be later fundamental in the composition of Beta existential graphs.

²⁴ In Peirce's analysis, signs are always *triadic*. A first level of triadicity is found in the very definition of sign as a *ternary generic relation* $S(-, -, -)$: 1 substitutes 2 for 3. Term «2» is the «object» of the sign; term «1», which substitutes the object, is its «representamen»; term «3» is the medium, the interpretation context, the «quasi-mind» where the substitution is carried; inside that quasi-mind, the representamen acquires a new form: the «interpretant». A second level of triadicity – sub-qualifying the three ways in which object and representamen can correlate – produces Peirce's well-known initial classification of signs: icon (1), index (2) and symbol (3). An icon substitutes a given object: it signals a syntactic mark. An index is an icon which, furthermore, detects some changes of the object: it signals a semantic variation. A symbol is an index which, furthermore, weaves variations along an interpretation context: it signals a pragmatic integration. All sorts of other sub-determinations are possible and the taxonomy can be *refined recursively*; Peirce came to distinguish at least 66 specific classes of signs. A «polymath semiotic combinatorics» is here in action.

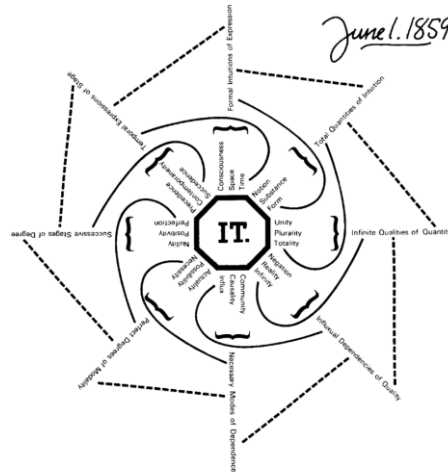


Figure 1.
Diagram of the It (1859)

The *movement* of the phenomenological categories will foster his work around relational graphs and composability of relations²⁵. Many of Peirce’s results remained just sketched, but McCurdy’s *Peirce’s Composability-of-Relations Theorem: A Proof in the Combinatorial Topology of the Logic of Relations* (2016)²⁶ introduces Peircean relational graph theory (PRGT), a new branch of topology which provides systematic foundations for Peirce’s ideas. PRGT studies the mathematics of relational graphs with exactly three kinds of relational vertices, motivated by ideas to model Peirce’s topological logic, Peirce’s semiotic systems, and certain results in quantum physics. Two main combinatorial operations – relative multiplication and autorelative multiplication – help to develop PRGT, yielding some strong generalizations of Euler’s formulas in classical graph theory and, especially, an extended version of Listing’s *census theorem*. McCurdy’s major result («Peirce’s Composability-of-Relations Theorem») states that to de/construct any kind of relational network in PRGT, it is necessary and sufficient to compose monadic, dyadic, and triadic vertices. This can be seen as an extension, and simplification, of Burch’s proof of Peirce’s *reduction thesis*²⁷, which states that thirdness is required in the *continuous combinatorics* of existential graphs, and cannot be reduced to a discrete combinatorics of units and couples, as happens in set theory (via Kuratowsky’s ordered pairs).

Murray Murphey’s *The Development of Peirce’s Philosophy* (1961)²⁸ remains, after more than sixty years, the best general guide to understand Peirce’s research in topology (and, beyond that, it still stands as the best global survey of Peirce’s entire

²⁵ Cfr. Peirce, *The New Elements of Mathematics*, cit., *passim*.

²⁶ W.J. McCurdy, *Peirce’s Composability-of-Relations Theorem: A Proof in the Combinatorial Topology of the Logic of Relations*, «Cuadernos de Sistemática Peirceana», 8, 2016, pp. 163-189.

²⁷ R. Burch, *A Peircean Reduction Thesis. The Foundations of Topological Logic*, Texas Tech University Press, Lubbock 1991.

²⁸ Cfr. Murphey, *The Development...*, cit.

work to date). Part three of the monograph – devoted to topology, geometry, number, and the classification of continua following Listing – offers a full panorama of Peirce’s *combinatorial topology* wanderings, in particular around cycloids, that is, the study of the genus of a surface (number of cuts, minus one, which disconnects the surface). Assessing in further detail Peirce’s topological writings, Jérôme Havenel’s Ph.D. dissertation (2006) extends Murphey’s first readings and offers the most precise particular guide to date of Peirce’s ideas on topology. Havenel’s *Peirce’s Topological Concepts*²⁹ sums up his main findings around Peirce’s *definition* of topology, Peirce’s reading of Listing and his extension of the *census theorem* $V - E + F - S = 0$ (for a given figure, V stands for the number of its vertices, E for edges, F for faces, and S for the disjoint spaces that conform the figure), Peirce’s work on the problem of *map-coloring*, and Peirce’s explorations around *dimensions*, motions, singularities, and defects of continuity. Havenel concludes that Peirce’s main topological insights were his understanding «that one could classify spatial complexes according to the value of their census number», his attention to «continuous structures useful for analysis (like lines, circles, disks, spheres, toruses, etc.)», and his combinatorial calibration of associated «topological singularities». All in all, we are thus in presence, not just of a logical *genius*, as underlined in *Sections 1* and *2*, but also of a mathematician of a high caliber³⁰. As we will see in *Section 5* below, this «polymath» inventive web (logical, mathematical, semiotic) helps to weave an extremely original philosophical view of the World.

4. Applied Mathematics

It may come as a surprise that a conceptual, abstract, general, universal mind, such as Peirce’s, was also extremely interested in practical applications³¹. Nevertheless, a glance at his material life shows Peirce working 32 years for the U.S. Coast Survey, where he devoted countless hours to pendulum calculations, photometric research, engineering mechanisms³². A *scientist before anything else*, as Jaime Nubiola has always underlined, Peirce’s «polymath scientific practice» oriented all of his work. Peirce’s correspondence around his European voyages (1870-1883), now edited by Barrena and Nubiola³³, shows an extremely busy young scientist, attentive to very practical

²⁹ J. Havenel, *Peirce’s Topological Concepts*, in *New Essays on Peirce’s Mathematical Philosophy*, edited by M. Moore, Open Court, Chicago 2010, pp. 283-322.

³⁰ Following Banach’s famous phrase around calibrations of mathematicians – «A mathematician is someone who finds analogies between *theorems*; a good mathematician sees analogies between *proofs*; a great mathematician sees analogies between *theories*. We can imagine the supreme mathematician seeing analogies between *analogies*» (cfr. R. Kaluza, *Through a Reporter’s Eyes: The Life of Stefan Banach*, Birkhäuser 1995, our emphasis) – one can consider Peirce to be a ‘good’ mathematician.

³¹ For this *Section 4*, and in general for many discussions around Peirce’s logic and mathematics, we are indebted to A. Oostra. In particular, his presentation on *Peirce y las Matemáticas* at his *Seminario Permanente Peirce* (17th April 2024 – Cycle 29, Session 4) provides much of the information included in this section.

³² Edison Torres has been preparing an edition of Peirce’s multifarious correspondence around engineering themes. We thank here Edison for his strenuous work.

³³ S. Barrena, J. Nubiola, *Los viajes europeos de Charles S. Peirce, 1870-1883*, EUNSA, Pamplona 2022 (English translation, De Gruyter, forthcoming).

matters related to varied «polymath» researches: mechanical, chemical, astronomical, geodesic. And many significant ulterior developments of Peirce's thought are engaged in *applied mathematics*:

- *foundations of statistics: Illustrations of the Logic of Science* (1877-1878)³⁴, *A Theory of Probable Inference* (1883)³⁵, work with Jastrow on random proofs (1885)³⁶,
- *geography/ cartography: Quincuncial Projection* (1879)³⁷
- *electric computers: Letter to Marquand* (1886)³⁸
- *aeronautics: A Problem in Aerodromics* (n. d.)³⁹ – observations to Langley.

In all of these endeavors, Peirce shows the true mark of a committed scientist, attentive to particular details, but always willing to insert them in the bigger picture of universal knowledge. The true force of a «polymath» vision lies precisely in this *pendulum* between the concrete and the general, the material and the conceptual, the real and the ideal. Peirce's *inventive web* covers extremely *pointed* contributions, inserted in multiple contexts, but in turn *structured* by large dialectics. It is this *back-and-forth* between the local and the global which insures the robustness of Peirce's architectonics. Specialties (either logical, mathematical, physical, semiotical) are used as experimental fields, which propel new orientations to the «polymath» generic fabric which envelops them. As we will see in *Section 5*, here is where Peirce's deepest visions pervade all the architecture: pragmatism, synechism, continuous semiotics, modal hierarchizations.

5. Local and Global Polymath Perspectives

We have seen how Peirce's «polymath vision» acts on a *complex hierarchy of epistemological layers*: logical (standard and nonstandard), mathematical (algebraic and topological, combinatorial and geometric, abstract and applied), semiotic (cenopythago-

³⁴ C.S. Peirce, *Illustrations of the Logic of Science*, edited by C. de Waal, Open Court, Chicago 2014. Particularly the third and fourth articles of the series, *The Doctrine of Chances* and *The Probability of Induction*, lay the foundations of inferential reasoning.

³⁵ Id., *A Theory of Probable Inference...*, cit.

³⁶ C.S. Peirce, J. Jastrow, *On Small Differences in Sensation*, «Memoirs of the National Academy of Sciences», 3, 1885, pp. 73-83.

³⁷ C.S. Peirce, *A Quincuncial Projection of the Sphere*, «American Journal of Mathematics», 2, 1879, pp. 394-396. For a complete presentation, see L. Solanilla, A. Oostra, J.P. Yañez, *Peirce Quincuncial Projection*, «Revista Integración Temas Mat.», 34, 2016, pp. 23-38.

³⁸ C.S. Peirce, *Letter to Marquand* (30 December 1886), in C.S. Peirce, *Writings*, Indiana University Press, Bloomington 1993, vol. 5, pp. 421-423. Marquand was one of Peirce's disciples at Johns Hopkins, he designed a new logical machine based on electrical circuits (1886), studied art history, and went to be a curator of Princeton's Art Museum. Polymath universality ran also in Peirce's heirs...

³⁹ Id., *A Problem in Aerodromics*, manuscript, n. d. Peirce offered historical investigations and provided mathematical calculations to support Langley's aviation prototypes, finally anticipated by the Wright brothers (cfr. Oostra, *Peirce y las matemáticas*, cit.).

rical and linguistic). One can then imagine Peirce's system as a *complex Riemann Surface*, with many intertwined sheets of knowledge. *Diversity and unity* are welded together in each of the *local* layers (logic, mathematics, semiotics), and become in turn reflected in the *global* architecture (philosophy). *Breadth and depth* offer an elastic and solid stronghold to the architectonics. The main arches of the system are related to *modalized continuity*: (i) *Pragmaticism* (1903)⁴⁰, where the Pragmatic Maxim (1878)⁴¹ is understood in modal terms, (ii) *Synechism* (1893)⁴², where continuity is multilayered over the whole universe, (iii) *Semioticism* (1886)⁴³, where sign iterations try to modulate the infinite range of interpretation. Aristotle's influence on Peirce in the 1880s cannot be underlined enough⁴⁴, yielding a comprehensive modalization of all his thought.

Through pragmaticism, synechism, and semioticism, signs are allowed to *contaminate*⁴⁵ each other around *borders*, something captured along Peirce's Continuum and *reflected* iconically in the Existential Graphs. The very layers of the architectonics allow to go *back-and-forth* between the local (inscription in the graphs) and the global (cultural irradiation along the continuum). This *back-and-forth* between generality and concreteness, with *infinite modal layers in the mediation*, pervades Peirce's architectonics. Towards maximality, completeness, and universal continuity, some *global* forces fill the space of knowledge, in a «topologically dense» sense. On another hand, an associated «density» of (global → local) codifications and (local → global) irradiations, secures the adequate complexity of the many cognitive layers involved (see *Figure 2*).

⁴⁰ «Pragmatism [later called Pragmaticism] is the principle that every theoretical judgement expressible in a sentence in the *indicative* mood is a confused form of thought whose only meaning, if it has any, lies in its tendency to enforce a corresponding practical maxim expressible as a *conditional* sentence having its apodosis in the *imperative* mood» (C.S. Peirce, *Harvard Lectures on Pragmatism* [1903; *Collected Papers* 5.18]; we emphasize the central modal terms: actuality, possibility, necessity).

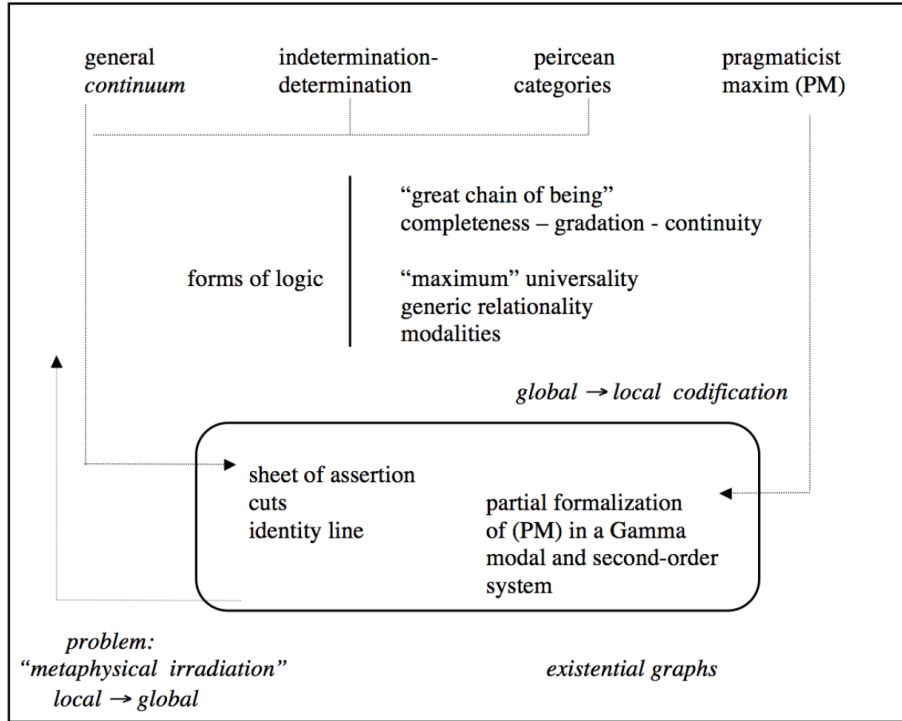
⁴¹ «Consider what effects which might conceivably have practical bearings we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object» (Id., *How to Make Our Ideas Clear* [1878; *Collected Papers* 5.402]).

⁴² «The word *synechism* is the English form of the Greek *συνεχισμός*, from *συνεχίζω*, continuous. (...) I have proposed to make *synechism* mean the tendency to regard everything as continuous. The Greek word means continuity of parts brought about by surgery. (...) I carry the doctrine so far as to maintain that continuity governs the whole domain of experience in every element of it» (Id., *Immortality in the light of synechism* [1893; *The Essential Peirce* 2.1]).

⁴³ «Every kind of sign, representative, or deputy, everything which for any purpose stands instead of something else, whatever is helpful, or mediates between a man and his wish, is a Third» (Id., *One, Two, Three: an evolutionist speculation* [1886; *Writings* 5.300-301]).

⁴⁴ M. Fisch, *Peirce's Arisbe: The Greek Influence in His Later Philosophy*, in Max Fisch, *Peirce, Semiotic and Pragmatism*, Indiana University Press, Bloomington 1986. Cfr. «From Epicurus's chance, for example, Peirce moved to the chance and spontaneity of Aristotle, and in general to Aristotle's logical and physical modalities in relation to his own categories», *ivi.*, p. 232.

⁴⁵ Against the usual depreciation of the word, «contamination» is here understood as a *positive* action, opposed to stringent forms of «purity» (e.g. «analytical»), and open to vague transits and *liaisons impures*: life itself.



Peirce's architectonics

Figure 2.
Global and Local Reflections in Peirce's Architectonics

Peirce's architectonics may be seen then as an *infinitely iterated surface*, with many connections between its layers⁴⁶. Each «polymath» sheet of the surface offers the possibility to explore *in depth* many particular specifications, but the *whole* surface itself (an iterated «polymath of polymaths») offers thousands of paths to run through an incredible *breadth* of knowledge.

Ringraziamenti. A Rossella Fabbrichesi, per il suo brillante esempio, la saggezza, la profondità, la tenacia.

⁴⁶ This is captured mathematically by the Riemann surface of the *complex logarithm*. Infinite copies of the complex plane are stacked and welded together in the Riemann surface of $\log(z)$.