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HUSSERL ON THE CONCEPT OF *ANZAHL*

Three Ways Not to Conceive it*

Rodolfo Castagnino

ORCID: 0009-0005-3197-9498

Università degli Studi di Milano (ROR: 00wjc7c48)

Contacts: rodolfocastagnino00@gmail.com

Abstract

This article addresses the concept of *Anzahl* (cardinal number) as understood by the early Husserl, who made it the foundation of his *Philosophy of Arithmetic* (1891). The discussion is developed through a comparative approach: after retracing the key features of the characterization of the cardinal number presented in that work, the paper examines the criticisms Husserl directs at three alternative theories – two of which are found within the *Philosophy of Arithmetic* itself, and one is presented in a lecture given in 1901. These comparisons provide an opportunity to appreciate the specificity of certain aspects of Husserl's philosophy of mathematics, which is centered on the acts performed by the subject. Moreover, they offer valuable insights for re-examining two recurring themes in *The Crisis of European Sciences and Transcendental Phenomenology* (1954): the problematic relationship between mathematical ideality and the *Lebenswelt*, and the preference for a mathematical practice centered on a symbolic mode of thought.

Keywords: Husserl, Philosophy of Arithmetic, Cardinal Number, Material Content, Formal Concept, Sign.

Husserl e il concetto di *Anzahl*
Tre concezioni erronee

L'articolo affronta il concetto di *Anzahl* (numero cardinale) per come inteso dal primo Husserl, che lo pose alla base della propria *Filosofia dell'aritmetica* (1891). La discussione viene sviluppata secondo un approccio comparativo: ripercorsi i tratti decisivi della caratterizzazione che nell'opera l'autore offre del numero cardinale, si discutono le critiche che egli rivolge a tre teorie alternative alla propria – due di esse contenute nella

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stessa *Filosofia dell'aritmetica*, una inserita invece in una conferenza tenuta nel 1901. Tali confronti offrono l'occasione per apprezzare la specificità di alcuni aspetti della filosofia della matematica dell'autore, incentrata sull'attività del soggetto; inoltre, costituiscono preziosi spunti alla cui luce rileggere due temi che ricorrono in *La crisi delle scienze europee e la fenomenologia trascendentale* (1954): la problematizzazione del rapporto tra idealità matematica e *Lebenswelt*, e la predilezione di una pratica matematica centrata su un pensiero eminentemente simbolico.

Parole chiave: Husserl, Filosofia dell'aritmetica, numero cardinale, contenuto materiale, concetto formale, segno.

1. Introduction

The present paper discusses some of the main features characterizing the concept of cardinal number according to Husserl: to do that, I consider some passages from the author's early writings (1889-1901) on mathematics and adopt a comparative method. In §2 I sketch some of the basic notions presented in the *Philosophy of Arithmetic*[1], particularly Chapters I, III, and IV: the following discussions will presuppose these initial clarifications. In §3 I look at Chapter II of the same work and consider Husserl's critical analysis of Lange's and Baumann's accounts of number; this debate is valuable since it offers especially clear insight into some key aspects of the author's conception of what cardinal numbers are and how they should be thought of. In §3.1 I propose a critical reflection on the advantages of developing a philosophy of mathematics which assigns a pivotal role to intentional acts, such as Husserl's case. In §4 I recall the treatment of Helmholtz's position contained in the appendix that concludes the first section of the *Philosophy of Arithmetic*: there, the author has the chance to show how a misconception of the nature of number concepts can undermine the adequate understanding of arithmetic. In §5, I consider a passage from Husserl's notes for his 1901 lectures in Göttingen, where an assessment of Dedekind's proposal to consider numbers as mental creations can be found: this confrontation is enlightening, since it indicates a crucial distinction between two essentially different kinds of concepts of numbers. Finally, in §6, a brief look forward into Husserl's latest work is provided, in order to show how some of the topics previously discussed recur in the author's philosophy and to justify the selection of passages here proposed.

[1] E. Husserl, *Philosophie der Arithmetik. Mit ergänzenden Texten (1890–1901)*, ed. by L. Eley, Martinus Nijhoff, Den Haag 1970 (*PdA* from now on); Eng. trans. by D. Willard, *Philosophy of Arithmetic. Psychological and Logical Investigations with Supplementary Texts from 1887-1901,* Kluwer, Dordrecht 2003 (*PoA* from now on).

2. The notion of *Anzahl* in the *Philosophy of Arithmetic*

First of all, some observations about Husserl's lexicon are in order. By the word '*Anzahl*' the author refers to the concept of finite cardinal numbers (two, three, four, and so on), for which, by the time the *Philosophy of Arithmetic* was published, Ernst Schröder had recently proposed the nowadays more common name of "natural numbers". Although Husserl takes the term *Anzahl* to be his first choice, he also regularly resorts to the somewhat vaguer "*Zahl*", the usual German translation for number: for our current purposes it is enough to say that *Anzahl* and *Zahl* are synonyms, and that Husserl pays special attention to using the former when some contraposition is being made between cardinal numbers and other types of number, such as the ordinals (*Ordinalzahlen*). The first section of the *Philosophy of Arithmetic* is devoted to clarifying three basic notions that lie at the bottom of any act of numbering, namely those of cardinal number, aggregate (mainly *Inbegriff*, *Vielheit* or *Menge*), and collective connection (*kollektive Verbindung*): since they will be at the base of our discussion, it is useful to briefly recall what is most notable in each.

In essence, Husserl's take is that cardinal numbers are the precise quantification of the elements involved in an aggregate, where an aggregate is a collection of objects made possible by virtue of a specific type of connection, called collective connection. What makes this form of connection possible is the fact that, unlike others, it does not depend on the nature of the connected objects. Consider the relation associating a color to the surface it inheres to, or that unifying a rose, a whole whose parts are a stem, leaves, petals, and so on: in both cases, we are confronted with relations that connect their elements after what is intuitively given in the represented contents themselves, that is, after what Husserl calls their primary contents. Instead, the collective connection aggregates elements regardless of their primary contents: in this case the relation is merely an extrinsic one, since it has to be traced back to the psychical act that freely connects the contents – and not to the intuited contents as such. This implies a further, significant, difference: while grasping the relations that make up a rose only requires to observe what is being given in intuition, the collective connection rests on the performance of a mental act that arbitrarily aggregates elements sharing no common primary contents, such as a feeling, an angel, the Moon, and Italy.

An aggregate is a group of elements resulting from the performance of such an act, or, analogously, held together by the extrinsic link of the collective connection. What is typical of the concept of aggregate is its indeterminacy, not only – as just seen – for what concerns the irrelevance of the nature of the elements it connects, but also for the number of elements it can include, since, in fact, there is no limit to the quantitative extension that an aggregate can undergo. We now have enough tools to understand why Husserl presents the essential form of an aggregate as follows: «A certain something [*irgend etwas*] and a certain something and a certain something, etc.»[2]. This formula is worth discussing: in it, three traits are crucial. First, naming each of the connected elements «a certain something» is meant to stress the irrelevance of their nature: they are included in the aggregate only insofar as they are objects on which a psychical act of collective connection has been performed. Secondly, the preposition «and» expresses the collective connection itself: both the preposition and the collective connection simply juxtapose elements in a paratactic fashion, without imposing any further ordering on them. Lastly, the «etc.» indicates the ideally open structure of the aggregate, which knows no maximum number as for its elements. Note, finally, that according to Husserl the notions of collective connection and aggregate are two sides of the same coin, for they both indicate a group of elements, although apprehended from two different paths: while the notion of aggregate focuses on the manifold nature of the group as such, that of collective connection captures its essential core, what makes it what it is.

It is at this point that *Anzahlen* arise. The concept of cardinal number removes the indeterminacy intrinsic to the notion of aggregate by quantifying it, i.e. by specifying the precise number of elements included in the considered aggregate. Assigning to an aggregate its corresponding cardinal number amounts to furnishing a determinate description of how many its elements are. This means that numbering objects allows one to specify the general concept of aggregate: sticking to the formula just discussed, it is now possible to remove the «etc.» from it and consequently to distinguish aggregates of the form «a certain something and a certain something», which are assigned the cardinal number two, from those of the form «a certain something and a certain something and a certain something», which are assigned with the cardinal number three, and so on. The distinction between the concept of aggregate and the concept of *Anzahl* is thus one of fineness of grain, in that this enables a richer description of a group of objects, which is seen no longer just as the result of a collective connection, but also as an aggregate specifically determined as to its amount. Indeed, according to Husserl, to make a fine criterion to determine whether a concept is a cardinal number it is precisely the possibility to employ it as a proper answer to the question asking «how many» (*Wieviel*) the elements of a given aggregate are.

One last remark has to be made concerning the concept of cardinal number. According to Husserl, presenting the notion of cardinal number in general

[2] *PoA* p. 84; *PdA* p. 80. Some of the passages I quote contain passages from other authors quoted by Husserl: for the sake of brevity, I refer the reader to Husserl's works, where all the necessary references can be found.

as a *concept* is partly misleading, since «*Anzahl*» is strictly speaking only a general *name*. As just said, cardinal numbers determine the quantity of elements in an aggregate, by specifying how many these are. This implies, however, that the only authentic concepts of cardinal numbers are those such as two, three, four, and so on, whereas "cardinal number in general" is, in fact, just a name to designate the entirety of them: in other words, if one – so to say – looks into the notion of *Anzahl,* nothing more can be found in it than the specific cardinal numbers, the only ones capable of actually quantifying the elements of an aggregate. The comparison established as an example by Husserl is particularly clear: what holds for "*Anzahl*" holds for the name "color" as well, which is nothing more than the designation of the single colors and can by no means be taken to be something over and above blue, red, yellow, green, and so on. If, however, it is still possible to speak reasonably, although in a somewhat weaker sense, of a *concept* of cardinal number in general, it is because each specific cardinal number shares two common features with its peers: the identity of their units, which are only «a certain something», and the sameness of the relation linking them, i.e. the collective connection. Only by virtue of such a close resemblance between the concepts two, three, four, and so on can the general name "cardinal number" be accepted as a concept too.

Once these key points in Husserl's account of number have been recalled, we can now turn to his critical assessments of some rival proposals: their discussion will offer the chance to elucidate aspects of what has been outlined so far.

3. Against Lange and Baumann: Where Numbers Do Not Stem From

Chapter II of the *Philosophy of Arithmetic* engages in a detailed critique of several alternative conceptions of numbers in contrast with the author's view: among others, Friedrich Albert Lange's and Julius Baumann's theories are discussed.

The main stance Husserl associates Lange with is the reduction of any representation, including those of numbers, to that of space. Instead of following Kant in relating number concepts with the pure intuition of time, Lange maintains that they too root in the representation of space and of spatial things:

> We originally receive each number concept […] in the form of a sensuously determinate image of a group of objects, whether they are only our fingers, or the buttons and spheres of an abacus[3].

[3] *PoA* p. 36; *PdA* p. 35.

Showing right from the outset of his treatment of this position that he finds it untenable, Husserl observes that the above quote sounds particularly faulty since it implies that «the familiar general concept of number appears as an individual phenomenon, as the sensuously determinate image of a group of spatial things»[4]. In fact, according to Lange numbers would be grasped in the «synthesis of spatial intuition»[5] through which spatial objects and their properties are apprehended.

The first serious accusation that Husserl draws against Lange is that of confusing representations whose contents have the feature, among others, of being spatially determined, and representations of contents *qua* spatially determined: there is little doubt that cardinal numbers are numbers of things, possibly things in space, but this does not imply that number concepts bear in themselves any essential reference to some spatial determination characterizing the counted objects. Even if numbering were possible only for spatial things – which is not the case –, Husserl states that

> this does not yet decide whether the representation of space nevertheless makes a special contribution to the *content* [*Inhalt*] of the number concept. It is easy to see that it does not[6].

To prove this, the author highlights the irrelevance, when it comes to counting, of the several configurations that two apples can assume: nearer or closer, irrespective of whether one is on the left or at the top of the other, and so on, they are counted as two apples nonetheless, and their place in a spatial synthesis is irrelevant.

By attempting to explain statements on numbers in terms of statements on space, Lange missed to acknowledge an essential heterogeneity between two layers of experience. According to Husserl, he failed to see that there exist syntheses that are possible where no internal connection between primary contents – ultimately, for Lange, space – can be found: «According to him [Lange] all combination is supposed to occur in the content, and of course in virtue of the form of space [*Raumform*] encompassing all content»[7]. This conviction prevented him from appreciating the specificity of the collective connection, which is independent of the features of aggregated objects: Lange could never acknowledge that unlike space, which is a material content of experience, number concepts

4 *Ibidem*.

5 *Ibidem*.

6 *PoA* p. 37; *PdA* p. 36.

7 *PoA* p. 42; *PdA* p. 41. Note that «combination» translates the "*kollektive Verbindung*" that I have so far rendered with "collective connection".

are formal categories[8]. This blindness is responsible for a major descriptive lack within Lange's theory: indeed, it cannot account for our ability to isolate different numbers of objects out of an identical synthetized content. Husserl considers the scenery where five things are counted, and writes:

> Can we not in the next moment pick out from among them merely two, three, or four, through a unifying act of interest, without in the least affecting the factually present combinations of contents (e.g., the distances, physical bonds, or the like)[9]?

Such a switch of interest most certainly can take place, but Lange is not able to explain why this is so: for it to be understood one must first realize that numbers and the collective connection they originate from emerge, once again, on a different level than primary contents, namely that of a psychical act.

The last remark of Husserl's refutation concerns the correct way to conceive synthetizing acts and their role: like Kant before him, Lange assumes them to be activities through which the corresponding unified objects would be *created.* Precisely the assumption that such objects are considered nonexistent prior to the act being performed is the reason why Husserl dismisses this interpretation:

> Our mental activity does not *make* [*macht*] the relations. They are simply there, and, given an appropriate direction of interest, they are just as noticeable as any other type of content. Strictly speaking, creative [*schöpferische*] acts that produce some new content as a result distinct from them are psychological monstrosities [*Undinge*][10].

A rigorous distinction is drawn here between the subject's mental activity and the external objects constituting its potential contents: the mind does not create anything outside it; rather, its acts are but the intellective conditions necessary for the reception of external objects. Starting from here, Husserl averts a possible accusation against his own theory of collective connection. Indeed, this connection too is a synthetic act, since it unifies elements into an aggregate: here the content of the act, i.e. the aggregate, cannot be traced back to primary contents by definition of the collective connection itself, and this could lead one to think of the latter as a creative act. However, this is not the case and no contradiction arises, because

8 As Claudio Majolino rightly noted, by drawing this distinction Husserl is already maintaining the position later to be defended in the *Logical Investigations* according to which geometry is an empirical science while arithmetic is a branch of formal logic (cf. C. Majolino, *Declinazioni dello spazio. Sul rapporto tra spazialità percettiva e spazialità geometrica nel primo Husserl,* «Paradigmi», 64-65, 2004, pp. 223-238, esp. pp. 233-234).

9 *PoA* p. 43; *PdA* p. 42.

10 *PoA*, pp. 43-44; *PdA*, pp. 42-43.

> The combination of course subsists solely and only in the unifying act itself […]. But there does not exist besides the act a relational content different from the act itself, as its creative result, which the view we are attacking always presupposes[11].

The act of collective connection actually is responsible for determining an object, i.e. its corresponding aggregate, which could never be found among the primary contents; but by no means does this imply the creation of the correlate of a psychical act: performing the act of collective connection only leads to a concept, the concept of number, which is the content of the act and nothing beyond it.

The nature of psychical acts is the focus around which Husserl's assessment of Baumann's position as well is centered. There are analogies between the latter's stance and that of Lange, whose account partly relies on Baumann's: he too misplaces numbers by positing their origin in external things, albeit he develops deeper insight into the role played by the subject in the process of their formation.

Husserl begins by considering Baumann's understanding that numbering objects is an act to be framed into the psychical activity of the mind, rather than the immediate result of their simple contemplation:

> Baumann […] affirms the participation of our psychical activity in the formation of the number concepts. He states, for example: "The grasping together of 1, 1, 1 in 3 is a novel act of the mind, incomprehensible to anyone who cannot *do* it. That is, the mere seeing of one and one and one thing still does not yield the number 3, but rather this novel grasping together requires first to be done." Thus arithmetical concepts in general […] originate through a "mental action that can be incited and apprehended only in inner intuition"[12].

Husserl praises Baumann for acknowledging the active role played by the mind in the conceptualization of number (*Zahlenkonzeption*), i.e. in the formation of the concept of number. It is thereby refuted any attempt to reduce the counting of objects to a merely reproductive activity of the mind, one that would just explicitly predicate the feature of "being-a-certain-amount of" a state of affairs already containing that feature as one among its primary contents. On the contrary, affirming that there are four nuts on the table requires more than just a «passive reception or a mere selective noticing of a content»[13]. As seen in §2, numbering requires as its first step the constitution (*Bildung*) of an aggregate out of given objects, and it is only after such an aggregate has been constituted that it becomes possible to assign

[11] *PoA* p. 44; *PdA* p. 43.
[12] *PoA* p. 45; *PdA* p. 44.
[13] *PoA* p. 46; *PdA* p. 45.

a number to it. The ability to delineate the boundaries of an aggregate consists precisely in the mind establishing a collective connection that includes some contents and excludes some others: hence, an active psychical intervention turns out to be crucial for any possible act of numbering. Husserl writes with great clarity that:

> Which […] objects, and how many of them, we colligate and enumerate depends solely upon our interest, and thus the unification of the colligated is exclusively determined and accomplished by a psychical act of the type described above [namely, an act of *kollektive Verbindung*][14].

The author is also willing to accept, with Baumann, that numbers are «in a certain manner, purely mental creations [*rein geistige Schöpfungen*]»[15]: without the psychological activity that extrinsically connects different objects, and thus remaining confined to the level of primary contents alone, no thought of numbers whatsoever would be possible.

However strong the agreement with Baumann on this initial point may be, nonetheless Husserl is forced to refute a further and somehow contradictory assumption that his adversary made. In fact, Baumann also maintained that «external experience, on the other hand, is supposed to "bear the mathematical in itself, independently of our mind"»[16]: by saying so, he aimed at solving the otherwise mysterious problem of how mathematical sciences can be so brilliantly applied to empirical phenomena. However, this claim seems to Husserl both to be false and to essentially undermine the so far correctly developed account of numbers as creations stemming from mental activity. Indeed, within the framework of an experience that is mathematical in itself, such as the one Baumann just evoked, the theoretical import of positing numbers as mentally originated concepts remains far from clear, let alone necessary: were numbers already outside us, why should it be necessary to think of them as mental creations? Therefore, Husserl attacks Baumann's internally incompatible position by coherently defending the stance that his adversary could take only clumsily:

> In the case of external activities, one certainly distinguishes the activity from the product which it generates and which can externally persist when the activity itself is long since gone. But the psychical activities which ground the number concepts certainly do not produce in them new primary contents which, cut loose from the engendering activities, could then be found again in space or in the external world[17].

[14] *PoA* p. 47; *PdA* p. 46; my insertion in square brackets.

[15] *PoA* p. 46; *PdA* p. 45.

[16] *PoA* p. 45; *PdA* p. 44.

[17] *PoA* p. 47; *PdA* p. 46.

The inconsistency of Baumann's proposal rests on the fact that admitting a mathematically structured reality is incompatible with the claim affirming the mental origin of the concept of number: if the external world is endowed with some mathematical design, then numerical concepts *ipso facto* have a grasp on some primary contents of experience, which is precisely what the author denies. Note that in this last quote, by distinguishing between external and psychical activities Husserl is supporting once more his opposition to Kant and Lange, who thought that synthetizing acts could create real objects: only external activities have such power.

According to Husserl, in accounting for numbers Baumann was essentially misled by «an erroneous conception of the abstraction process [*Abstraktionsprozesses*] that yields these concepts»[18]. The whole goal of the first part of the *Philosophy of Arithmetic* is to show how tightly related to our intuitions *Anzahlen* are, due to their roots in everyday experience; nonetheless, there is no way one can – so to say – stumble across them, as if they were colors, sounds, or physical objects, because for numbers to be grasped a psychical activity is required:

> That which is intuitively present which we can encounter and observe in space, certainly does not consist of numbers in and for themselves, but consists, rather, only of spatial objects and of their spatial relations. But with that no number is yet given […]. The co-existence of objects in space is still not that collective unification in our representation which is essential to number[19].

Baumann fails to distinguish between *single* counted objects and the collective connection: only the latter is to be held responsible for the possibility to conceive those objects as *an* aggregate, *a* plurality of objects, and hence for the rise of the concepts of cardinal numbers, which are but specifications of the concept of aggregate by quantification. The distinction that Baumann misses is made possible thanks to the aforementioned abstraction: through it, one can acknowledge that numbering processes, although counting objects, are never centered on objects *as such,* since these are taken to be nothing more than «a certain something»[20].

3.1. *Critical Remarks*

To conclude this paragraph, I would like to stress the strength that Husserl's philosophy of mathematics gains by emphasizing the role played by intentional acts,

18 *PoA* p. 46; *PdA* p. 45.

19 *PoA* p. 47; *PdA* p. 46.

20 For a brilliant study on the role of abstraction in the *Philosophy of Arithmetic,* see P. Spinicci, *Astrazione e riflessione nella «Filosofia dell'aritmetica» di Edmund Husserl*, «Rivista di Storia della Filosofia», 42, 3, 1987, pp. 519-537.

of which Husserl provides a careful investigation and which lie at the core of his confrontation with Lange and Baumann.

In the first place, the debate considered so far shows the advantage of developing a philosophical theory of number which includes no reference to the peculiarity of the single counted objects. What makes Lange's proposal significant is its attempt to found number concepts on the material components of experience, such as space, and the intuitions related to them. On the other hand, in §§2 and 3, I stressed how crucial it is for Husserl that in his own account no connection can be established between what is actually given as a primary content of experience and numbers themselves. Hence, the advantage of Husserl's thesis over Lange's – and to some extent over Baumann's too – is a descriptive one: only his theory can justify the undoubtable fact that we are able to count *any* object entering the field of our experience. Indeed, by refusing that the concept of number refers to, or even depends on that of space, the author can justify the fact that we can number objects devoid of spatial extension, and even collect groups of objects among which some have a location in space while others occupy no place at all.

However, a deeper assessment can be given of the advantages taken by a theory of mathematical knowledge centered on the phenomenological notion of act. Nowadays, it is quite customary to label Husserl's position regarding mathematical objects as a form of Platonism, the theoretical option according to which mathematical statements revolve around *existing* abstract objects. Indeed, little doubt there can be about the fact that already since his *Philosophy of Arithmetic* Husserl acknowledges the existence of mathematical objects, such as cardinal numbers; however, a further step is in order for gaining more refined insight into Husserl's conception of mathematical objects[21].

While introducing the Platonist stance in mathematics, Marco Panza and Andrea Sereni stress the tight bond relating i) the existential statement about an abstract mathematical object, ii) the clarification of the nature of such an object, iii) and consequently the way in which it exists: merely asserting that numbers exist (i) may not mean much if no further details are provided on what they are (ii) and, hence, on how their existence is to be understood (iii)[22]. Moreover, mathematical Platonism faces another challenge: it is compelled to tackle the

[21] «*Husserl sans doute est 'réaliste' en logique, comme se plaisent à l'écrire les commentateurs anglo-saxons; mais le difficile à comprendre est qu'ille soit toujours déjà sur le terrain – et sur le terrain seulement – de la phénoménalité*» (J. Benoist, *Phénoménologie, sémantique, ontologie. Husserl et la tradition logique autrichienne*, Presses Universitaires de France, Paris 1997, p. 233).

[22] Cf. M. Panza, A. Sereni, *Il problema di Platone. Una storia della filosofia della matematica e un'introduzione al dibattito contemporaneo,* Carocci, Roma 2009, pp. 21-22. Note that only points i) and ii) are discussed there: I added point iii) since I believe it to be implicitly contained in ii).

so-called "access problem", that is to explain how abstract and existing mathematical entities are known[23]. Without committing to too detailed an exposition, the rest of this subparagraph is devoted to sketching how Husserl's philosophy fits into this debate: his proposal proves a sound attempt to make sense both of mathematical concepts and of mathematical knowledge by insisting solely on the role played by acts of consciousness. Since the route undertaken in the *Philosophy of Arithmetic* culminates in the Logical Investigations, these are worth considering in order to fully appreciate Husserl's theory of mathematical objects.

The *Second Logical Investigation* is devoted to clarifying the consciousness of general objects (*Allgemeinheitbewusstsein*), among which mathematical ones are included, and to presenting the peculiar form of Platonism or Idealism explicitly endorsed by the author. In the introduction, he states that his goal is now

> to assure the basic foundations of pure logic and epistemology by defending the intrinsic right [*Eigenberechtigung*] of specific (or ideal) objects to be granted objective status alongside of individual (or real) objects. This is the point on which relativistic, empiricistic psychologism differs from idealism, which alone represents the possibility of a self-consistent theory of knowledge[24].

To begin with, Husserl's Platonism has a broader meaning than the mathematical one, since it is thought of as a global theory of species and general objects: it acknowledges as existent not only – say – numbers but also other general concepts, such as the red color. Hence, in Husserl's philosophy, mathematical Platonism appears as the application of a general strategy, that of recognizing the existence of general objects, to the particular domain of mathematics[25]. Once this clarification has been offered, it is true that the author has no doubt about the lawfulness of defending i): refusing to accept the existence of general objects

23 «*Se gli esseri umani posseggono cinque sensi per osservare la realtà materiale che li circonda, con che mezzo essi osservano la realtà matematica? Se fosse possibile risolvere facilmente questo problema, oggi detto 'problema dell'accesso', il platonismo in filosofia della matematica sarebbe probabilmente una tesi scontata. Ma così non è. È per questo che, pur essendo un'opzione ontologica, il platonismo porta con sé un problema epistemologico che potremmo, in generale, formulare così: se la matematica parla di oggetti astratti, come possiamo conoscere ciò di cui parla?*» (M. Panza, A Sereni, *Il problema di Platone,* cit., p. 38).

24 E. Husserl, *Logische Untersuchungen. Zweiter Band, erster Teil*, ed. by U. Panzer, Kluwer, Dordrecht 1984, p. 112 (*LU* from now on); Eng. trans. by J.N. Findlay, ed. by. D. Moran, *Logical Investigations,* Routledge, London 2001, p. 238 (*LI* from now on). Husserl's analogous self-introduction as a logical Platonist occurs in his attempt to write a new preface to the second edition of his *Investigations*: it is pointed out below how loosely the author's usage of that labelling is.

25 This is the picture ultimately emerging from the *Logical Investigations*' theoretical framework. However, such a picture does not contrast with the legitimate assertion that Husserl's conceptions of the *Allgemeinheitbewusstsein* and general objects was at first developed precisely starting from reflections on the nature of mathematical objects and truths: see e.g. Husserl's own statement opening the *Preface* to the first edition of the *Investigations*.

would be a serious misunderstanding of what and how our consciousness' activity unfolds – and ignoring this amounts to betraying the descriptive attitude at the core of phenomenology.

As for ii) and iii), it is legitimate to think that – at least in the *Second Logical Investigation* – Husserl's attention is mainly devoted to tackling the issues they raise. Starting from iii), the author stresses that his idealistic position in logic implies no metaphysical commitment. In particular, and having Plato as his critical target, he refuses to identify his own claim on the existence of general objects «with the assumption that the Species really exists externally to thought»[26]. Acknowledging the existence of general concepts such as the color red and number two does not amount to asserting that these exist in a real (*real)* fashion, i.e. in space and time, the way a book or a human being do. Rather than ending in this erroneous hypostatization, which conceives existence only as factual existence, Husserl embraces the notion of holding (*gelten*), that is of being valid – or existing – *ideally*, i.e. regardless of any temporal determination[27]. This appears as the only plausible way to conform to what is given: on the one hand, as seen in i), the existence of general objects cannot be denied without losing a grip on our experience; on the other hand, the tenet of a philosophical description of our experience could not tolerate the forced conflation of two different modes of existence into a single one. Hence, the answer to iii) is that general objects exist in virtue of their validity and have no connection with spatiotemporal determinations.

For the present reading, the answer to ii) constitutes the most characteristic trait of Husserl's position. Usually, mathematical Platonism is immediately taken as a strong defense of the existence of mathematical entities, and it is therefore considered as a realist reflection on mathematical objects. However, such an approach is inconceivable within the metaphysically neutral framework of the *Logical Investigations*, which, on the contrary, tackles the issue of ideal objects a parte subjecti. In other words, from their merely descriptive point of view, the only relevant datum consists in the intentional life of consciousness: there is no place for objects whatsoever unless their constitution through performed intentional acts has first been secured. If then one looks at intentional acts only, these reveal that often numerically different acts are directed toward the same meaning: their meaningfulness involves a reference to an invariant element that can be taken to mark their ideal unity[28]. Indeed, Husserl's Platonism comes into play

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[26] *LI* p. 248; *LU* p. 127: «*Die Annahme einer realen Existenz von Spezies außerhalb des Denkens*».

[27] Here Husserl benefits from the reading of Plato's doctrine of ideas that Lotze developed in his *Logic*: for the sake of brevity, this topic will not be dug into here.

[28] «*L'invariant est ce qui est produit dans la conscience* […] *que c'est 'le même' qui est exprimé, au sens du 'même' qui serait exprimé dans les autres occurrences du même acte d'expression. L'unité de significa-*

as the answer to the descriptive fact that we can mean the same thing repeatedly: it originates from reflections on intentionality and language rather than from a realist ontology[29]. Once Husserl's conception of general objects has been understood, ii) can be correctly answered. Mathematical "objects" are but the identical goals intended by the intentional acts performed by consciousness when thinking about mathematics: their ideal nature and their generality follow from the peculiar characters of the acts that direct consciousness toward them – characters whose illustration is developed in the *Second and the Sixth Logical Investigation*. Furthermore, by choosing consciousness' intentional acts over objects as a starting point to found his idealistic stance in logic, Husserl can avoid giving an ontological treatment of the access problem, which is bypassed. Indeed, our access to mathematical objects need not be *explained* by introducing some sort of connection between consciousness and another alleged ontological dimension to which those would belong. Rather, mathematical and general objects are the meaningful ideal units aimed at by the intentional acts performed by consciousness when thinking about the red color or number two, i.e. by the *Allgemeinheitbewusstsein*: the simple act of intending them provides access to those ideal objects, which would not enter the field of conscious experience otherwise.

4. Against Helmholtz: What Numbers Are Not

In Husserl's early reflections on arithmetic, Helmholtz is one of the most prominent figures to be often mentioned and whose contribution is dismissed as a distorted understanding of what numbers are. Two main claims can be indicated as the reasons for such dissent: first, the nominalist position Helmholtz assumed regarding arithmetic; second, his claim that the cardinals could only be conceived after and by means of the ordinals. Both these issues are addressed by Husserl in the appendix placed at the end of the first part of the *Philosophy of Arithmetic* and titled *The Nominalist Attempts of Helmholtz and Kronecker* – incidentally, very little is said there about Kronecker.

tion est l'idée d'une identité de visée» (J. Benoist, *Phénoménologie, sémantique, ontologie*, cit., p. 54).

29 Emiliano Trizio pointed this out with great clarity when he stated that Husserl's idealism in the *Logical Investigations* «*È soltanto una conseguenza generale della verità di certi giudizi, 'di quelli cioè nei quali si giudica sui numeri, le proposizioni, le figure geometriche, ecc.' e* […] *si configura come una presa di posizione ontologicamente minimale derivante dalla necessità di ammettere la sensatezza e la verità di certe classi di giudizi. Si tratta quindi di un idealismo dettato da considerazioni logiche e gnoseologiche, e che pertanto deve trovare il suo chiarimento ultimo nella fenomenologia dei vissuti logici e, in particolare, nella fenomenologia delle forme della coscienza della generalità*» (E. Trizio, *Gli oggetti generali tra ontologia, logica e fenomenologia. Commento alla* Seconda ricerca logica, p. 108, in D. Manca, F. Nobili (a cura di), *Le* Ricerche logiche *di Husserl. Un commentario*, ETS, Pisa 2024, pp. 99-115). Note that the same entanglement between epistemological and ontological issues was already detected in the passage quoted in footnote 23.

According to the author, Helmholtz fell into the same misconception that also afflicted Berkeley, namely that of reducing numbers to the signs used to designate them, thereby ignoring their true conceptual nature. Numerals, i.e. symbolic notations indicating the concepts of numbers, and numbers themselves are here identified with one another. Once numbers are thought of in this way, it is easy to deem them «first and foremost […] arbitrary symbols [*willkürliche Zeichen*]»[30], as Helmholtz defined them indeed: this is what Husserl calls nominalism. Since his adversary did not go to great lengths to clarify «what it then is that these symbols do genuinely *signify*»[31], Husserl himself endeavors to find it out. It is worth recalling here that the whole second section of the *Philosophy of Arithmetic* rests on the conviction that signs serve as substitutes *for* something else, i.e. the symbolized: it could well be the case that what is substituted by them is either only temporarily or on principle unavailable to consciousness, but the fact remains that the necessary role played by the substituted must be acknowledged for the notion of symbol to make sense. Therefore, there must be a reference corresponding to the sign, i.e. a unity to which any numeral corresponds and which justifies its use:

> In the different cases they [symbols] can designate the most heterogeneous of objects, and yet the designation of those objects is no arbitrary one. Wherever we use the term «five», it occurs in *the same sense* [*in demselben Sinne*]. In what is it therefore grounded that the most dissimilar of representational contents are designated in the same sense by these signs? In short, what is the *concept* which mediates [*vermittelt*] each use of the signs and constitutes the unity of their *signification*?[32]

If the attribution to a group of objects of its corresponding number is not to be random, there must be a criterion underlying it: «In the things or the group itself there must be found something that is specifically touched upon by these signs»[33]. The attribution of numbers cannot depend on the primary contents of the enumerated objects, since one can enumerate any group of things whatsoever. It cannot be arranged according to the fact that each enumerated object is a unity either, for this would not explain the variety of numbers. Thus, according to Helmholtz, the criterion guiding the enumeration process is to be found in the ordering principle numerals are endowed with by stipulation: once the succession of numbers – taken to coincide with numerals – has been stipulated, then

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30 *PoA* p. 182; *PdA* p. 173. Note that Husserl does not really distinguish between "sign" (*Zeichen*) and "symbol" (*Symbol*), and that he sometimes names symbol both the sign and the object the sign stands for; I will interchangeably use sign and symbol, for this does not imply ambiguities for the present purposes.

31 *Ibidem*.

32 *Ibidem*.

33 *Ibidem*.

numbering is but the process through which the elements of a group are ordered in a series. Now «each sign is a sign of an order, it is the sign of an *ordinal number* in the usual sense of the phrase. The *signification* of each sign lies accordingly in its *place value* [*Stellenwerte*]»[34]. Ultimately, numerals are signs that stand for the position of the enumerated objects: this is the concept that mediates their application and that constitutes the unity of their signification, as Husserl's phrasing went. However, a relevant shift should be clear by now, since numbering comes here to mean ordering: it would not be answering the question asking «how many» anymore, rather it would be the reply to the question asking "which place in the series". In other words, Helmholtz «confuses the concepts *one*, *two*, *three*, etc., i.e., the number concepts in the *common* sense of the word, with the ordinal number concepts (*first*, *second*, *third*, etc.)»[35].

In fact, Helmholtz deemed ordinal numbers to have pre-eminence over the cardinals, for the former would be defined through the latter. Take a group of objects, say *M*, whose ordering requires the sequence of numerals from 1 up to *n*: the number of objects included in that group, i.e. the corresponding cardinal number, is *n*. From this definition, Helmholtz observes that cardinals remain unchanged if the ordering of the objects that they number varies: however these may be reordered, thereby assuming different ordinals each, a unique cardinal will always correspond to them[36]. Husserl proves the untenability of such a theory by invoking a descriptive account of the usual understanding of statements about aggregates:

> If I say, e.g., "the number of these apples is four", I certainly do not then have in mind the circumstance that, given some ordering of the apples, the last element is the fourth, but rather precisely that one and one and one and one apple is present[37].

The content of *Anzahl* bears no reference to an ordering relation between the contents it numbers; instead, it involves exclusively the reference to their collective connection and the thereby founded aggregate that it quantifies[38]. Against Helm-

34 *PoA* p. 183; *PdA* p. 174.

35 *Ibidem*.

36 For a discussion of Helmholtz's theory of numbers, see F. Biagioli, *Space, Number, Geometry from Helmholtz to Cassirer,* Springer Cham, Switzerland 2016, esp. pp. 81ff.

37 *PoA* pp. 184-185; *PdA* pp. 175-176.

38 By the time the *Philosophy of Arithmetic* was published, Husserl had already gained full awareness that the ordinals and the cardinals constituted two essentially different kinds of number concepts, entertaining no relation – let alone a dependency relation – one with the other. This emerges clearly from the manuscript known as *Arithmetik der Reihen und reihenartigen Grössen*, almost certainly meant for the *Philosophy of Arithmetic*, unless later cut: the text is contained in E. Husserl, *Studien zur Arithmetik und Geometrie Philosophie der Arithmetik (1886–1901)*, ed. by I. Strohmeyer, Martinus Nijhoff, Den Haag 1983, pp. 154-214; for a philological and philosophical discussion, see C. Ierna, *The Beginnings of Husserl's Philosophy, Part 1: From* Über

holtz's foundation of the cardinals on the ordinals, Husserl rather maintains that these two classes include concepts whose content is altogether heterogeneous: the cardinals quantify aggregates, the ordinals order their elements.

So far Husserl has rejected Helmholtz's attempt to found the cardinals on the ordinals. However, the issue of nominalism has not been seriously tackled yet. In this respect, after stating that numbers are but arbitrary symbols, Helmholtz argued that nothing in the series of numbers could be said to be «natural», since it is just the result of an arbitrary convention. Husserl's reply is that such a claim holds exclusively for numerals: of course, other than a stipulation, there is no reason why «4» could not be taken to designate number three instead of «3». However, this is as much as can be granted to Helmholtz:

> Those who speak of a natural ordering in the domain of numbers surely do not mean the ordering of arbitrary symbols, but rather of certain concepts designated by means of them. Whichever we consider, whether the ordinal numbers or the cardinals […], we always come to the result that the sequential order is one grounded through the nature of these concepts themselves[39].

As soon as one acknowledges that numerals are but signs that stand for the corresponding number *concepts,* then any talk of arbitrary stipulations must cease: according to Husserl, neither the ordinals nor the cardinals are established by convention, because their succession depends directly on their conceptual content. It is because an aggregate of four elements is greater than an aggregate of three by one unit that a certain ordering between numerals is strictly necessary; analogously, it is because what comes fourth succeeds immediately, i.e. without leaving any empty place between them, what comes third that a certain ordering between numerals is strictly necessary.

Husserl identifies the reason for this error in a superficial understanding of the processes through which we enumerate. Indeed, when numbering a group of elements, we hardly ever pay attention to the fact that – say – by adding one further apple to the group of apples we are enlarging the previous aggregate by one unit. Rather, we compute on symbols, whether they are written or oral:

> We proceed in such a way as to correlate mechanically [*mechanisch zuordnen*] the number names with the members of the group to be counted, and then take the last name required as that of the number sought. In actuality the names serve us in the first place as a mnemonically fixed sequence of symbols devoid of content [*inhaltsleerer*]; for during the enumeration

den Begriff der Zahl *to* Philosophie der Arithmetik, «The New Yearbook for Phenomenology and Phenomenological Philosophy», 5, 2005, pp. 1-56.

[39] *PoA* p. 185; *PdA* p. 176.

> their conceptual content [*begrifflicher Gehalt*] is totally absent from our consciousness. Only after completion of the process, and in the light of its true purpose, does the […] number concept enter into consciousness as the signification of the resultant number word[40].

Provided that to each counted element a corresponding sign is associated, the numbering process unfolds without our mind being aware of it: it is only when it comes to *knowing* how many elements have been counted so far that consciousness cares about actually reaching the meaning, i.e. the concept, the last employed symbol stands for. For he neglected this dynamic and confused «symbol and thing»[41], Helmholtz could not see that behind the external and blind process of numbering there always lie the concepts for which the symbols involved in the process stand for.

Husserl's confrontation with the nominalist position is particularly relevant because the latter is antithetical to the explanation the author himself will provide for numerals in their relation to concepts. Indeed, the second section of the *Philosophy of Arithmetic* is devoted, among other tasks, to grounding the claim that any operation on symbols can be potentially accompanied by its counterpart on the level of actual inferences between concepts, and that in fact its legitimacy relies entirely on this possibility. In other words, Husserl holds that arithmetic would be inconceivable if there were nothing more to it than a mere game of signs: the possibility to blindly derive signs from other signs instead of performing actual judgments on number concepts strictly depends on the possibility that, in principles, any legitimate manipulation of signs can be justified by resorting to the concepts the involved signs stand for.

5. Against Dedekind: What Cardinal Numbers Cannot Become

The last contrast that I would like to assess is discussed by Husserl in the so-called "*Doppelvortrag*" he held in Göttingen during the winter of 1901: even without reconstructing here the bigger picture of the lectures into which the passages I will discuss fit, some preliminary considerations are necessary.

Already in his early writings devoted or related to the *Philosophy of Arithmetic,* Husserl deemed the problem of widening the numerical field as one of the utmost urgency for a philosophical understanding of mathematics. While no doubt could be raised about the legitimacy of the notion of *Anzahl,* which deserves full citizenship in the realm of concepts by virtue of its origin in every-

40 *PoA* p. 186; *PdA* p. 177.

41 *Ibidem*.

day experience, according to the Husserlian view – which in this respect is no exception within the second half of the nineteenth century – the same could not be said for other numbers, such as negative, rational, irrational numbers, and so on. In particular, these are philosophically suspicious notions because it does not seem possible to associate with them any concrete phenomenon from which they could stem: what is the empirical ground on which the square root of a negative number could be justified? There is none. This deficiency leads Husserl to name such notions "*Quasi-Zahlen*" – literally "almost-numbers" – thereby stressing the gap that separates them from the *Anzahlen,* whose rooting in intuition instead is guaranteed, as the first section of the *Philosophy of Arithmetic* certifies. The relevance of attempting to provide a philosophical understanding of *Quasi-Zahlen* is enhanced by the fact that their unclear status is nonetheless matched by the pivotal role they play in the vast majority of calculations: unclear as they may be as to their meaning, *Quasi-Zahlen* prove formidable tools for solving mathematical problems.

In this context, the two Göttingen lectures raise the issue of how computations involving *Quasi-Zahlen* can be deemed legitimate: this is what Husserl introduces here as the problem of the "*Imaginäres*" in mathematics, where '*Imaginäres*' is just another expression to designate almost-numbers. The answer provided by the author is that such computations can be deemed valid provided that some precise logical conditions are respected. However, rather than in Husserl's specific answer to the main problem just sketched, here I am only interested in discussing the author's critique of a theoretical move made by some mathematicians when it comes to defining how numbers could be understood[42].

Since almost-numbers are encountered as derivative notions from cardinals, their clarification soon translates into an inquiry into how the numerical field can be widened, or, analogously, how new number concepts can be introduced. Husserl discusses five different proposals to accomplish such an extension and consequently to compute in the *Imaginäres*: among them, one treats numbers as mental creations. According to this position too it is true that *Quasi-Zahlen* are

> concepts to which no object can correspond. But who forces us to stay within the restricted number domain? Numbers, after all, are mere creations of our mind [*Schöpfungen unseres Geistes*] through the act of counting[43].

[42] For an exhaustive assessment of Husserl's lectures see S. Centrone, *Logic and Philosophy of Mathematics in the Early Husserl*, Springer, Dordrecht 2010, pp. 148ff.

[43] *PoA* p. 413; *PdA* p. 434.

The possibility of exhibiting an object is not deemed to make a fine criterion to determine whether something is a number or not. Rather, this position argues that numbers are legitimate whenever a system of laws governing their behavior can be provided such that: i) some restrictions on the original numerical field are removed, i.e. the original numerical field is widened, and ii) the laws holding for the original numerical field keep holding even for the widened one. Husserl's example will help clarify these conditions.

Take the equation 'a + x = c': if c < a, then there is no solution for it within the realm of *Anzahlen*, the original numerical field. However, the solution can easily be found through the *definition* of a new number, number c – a, whose introduction is meant precisely to overcome the restriction on the possibility of performing subtraction. In this case, according to the discussed theory, the original numerical field would have been extended from the cardinal numbers to the negative numbers. As for i), such an extension amounts to the removal of some previously established restraints, namely the subtraction operation is no longer defined only where the minuend is greater than the subtrahend. As for ii), the newly imposed rules do not conflict with those holding in the original field: this requirement of conservativity prevents the definition of negative numbers from being incompatible with the cardinals, since it is possible to compute through the extended field according to the same rules. If i) cannot be strictly called a condition on the creation of new numbers – in fact, it is the aim of the creation –, particular attention must be paid to ii):

> We have only to convince ourselves that the laws of operation for this number [the newly created one], which are carried over from the numbers defined as primordially valid and possible, can yield no contradiction in the total system of the operations[44].

This position was embraced by Richard Dedekind, whom Husserl explicitly quotes referring to his *Continuity and Irrational Numbers*: there one can read of the «novel act of creation [*Schöpfungsaktes*]»[45] through which the human mind brings to light new numbers meant to be the results of operations performed beyond initially given restrictions. Note that here the characterization of numbers as mental creations (*Geistschöpfungen*) acquires an altogether different meaning from the one attributed to it by Baumann. In the context of that discussion, referring to numbers as *Geistschöpfungen* amounted to acknowledging their dependence on a psychical act of constitution and to emphasizing the hiatus be-

44 *PoA* p. 414; *PdA* p. 434; my insertion in square brackets.
45 *Ibidem*.

tween those and concepts of primary contents, i.e. contents located in the external world; provided that the expression was understood in this sense, Husserl could then endorse the qualification of numbers as mental creations. On the contrary, I will now show why the author cannot accept numbers to be mental creations in the sense that Dedekind conferred on this expression, and that is taken by Husserl as potentially ending up in an empty verbal game.

After this minimal depiction, Husserl clearly states that the Dedekindian position is untenable, and to show why, he recalls the results of the analysis he developed ten years earlier in the *Philosophy of Arithmetic*. *Anzahlen* were there found to be the possible specifications of the concept of aggregate, with respect to the quantity of elements collectively connected; consequently, the most effective criterion for recognizing whether a concept was an *Anzahl* or not turned out to be checking whether it could serve as a proper answer to the question asking "how many". This reveals that the concept of cardinal number has a material content, meaning that it is bound up with an objectual field, i.e. that of aggregates of elements and their classifications in quantitative terms. Such a characterization, however, leads to acknowledging that the concept of cardinal number is «the *closed* manifold of particularizations that are possible in the sphere of the concept how many»[46], hence implying the impossibility of any extension of it. More precisely, widenings of the numerical field by creation, such as the one seen in the example, are far from guaranteeing that as a result of the creation a valid concept is obtained. This is the relevant point at stake: if, on the one hand, it is possible to define new numbers by modifying at a purely formal level the conditions for executing operations, on the other hand such a definitional procedure cannot be taken to ensure a corresponding match in terms of the meaningfulness of the concepts involved. Husserl writes:

> Now I certainly can give various definitions on the basis of the operations which are grounded in the Idea of the cardinal number. But certain results of operations are contradictory to the Idea of "how many"; and if I define these, then I have defined, precisely, contradictory numbers [*widersprechende Zahlen*]. The sphere of the concept of cardinal number I cannot, without absurdity, arbitrarily expand on the basis of creative definitions, for it is this concept, indeed, which imposes limits [*Grenzen*] on me[47].

Particularly the last sentence just quoted is here significant, since there lies the core of Husserl's argument against Dedekind's position. Meaningful material

[46] *PoA* p. 414, my emphasis; *PdA* p. 434: «*Die geschlossene Mannigfaltigkeit von Besonderungen, die in der Sphäre des Begriffes Wieviel möglich sind*».

[47] *PoA* p. 414; *PdA* pp. 434-435.

concepts do not allow for *any* manipulation, precisely in virtue of their meaningfulness: their material content prescribes them their rightful uses and prevents the wrongful ones, setting rigorous limits to the use that can be made of them. This is the kind of reasoning behind Husserl's distinction between *Anzahlen* and *Quasi-Zahlen:* while the former actually have a meaning, being justified on the ground of experience as valid answers to the question "how many", the latter are the results of merely formal stipulations, i.e. of definitions obtained by adjusting the laws regulating operations.

Husserl insists particularly on the arbitrariness of the characterization of numbers as definitions:

> The definition is an arbitrary stipulation [*Festsetzung*] of the signification of a word: In this we are certainly unrestricted. But once a word – e.g., the word "number" – is confined to a given domain of objects [*Objektgebiet*], one that clearly presents itself as possible, then I cannot decree through some sort of arbitrary stipulation that the domain shall admit of an expansion by means of new objects[48].

This opens up a deeper understanding of the issue Husserl is dealing with. As mentioned, *Quasi-Zahlen* pose a double problem: not only is their nature or meaning unclear from a conceptual point of view, but also their effective use in computations is mysterious, since it is not easy to see how notions with such an uncertain status can still prove crucial tools for arithmetical calculations. This last side of the question, namely that of *justifying* the successful recourse to such notions, remains totally unanswered by the Dedekindian approach. When it comes to trying to make sense of almost-numbers and to understand how and why they can provide correct results, merely stating that they should be thought of as definitions compatible with computational rules is not an answer, in that it does not explain the reason why their introduction should be understood as a theoretically safe move to be accepted as legitimate. Precisely because at stake here is an issue of justification, Husserl can state that «it is incomprehensible how one can claim that the difficulty is in some way eliminated by means of arbitrary definition [*durch die willkürliche Definition*]»[49].

As a conclusive remark, Husserl observes that the proposal discussed so far rests on «a certain conceptual displacement»[50], namely the shift from the concept of cardinal number to that of positive integer number: this precious hint is worth delving into. Once it has been made clear that *Anzahlen* can undergo no extension, Husserl writes:

48 *PoA* p. 414; *PdA* p. 435.

49 *Ibidem*.

50 *PoA* p. 415; *PdA* p. 435.

> But we no doubt can abandon the concept of number and, by means of the formal system of the definitions and operations that are valid for cardinal numbers, define a novel, purely formal concept, that of the positive whole numbers. And this formal concept of the positive numbers can, just as it itself is delimited by definition, be expanded by new definitions, and indeed in a manner free of contradiction[51].

A distinction is drawn here between two different kinds of concepts: on the one hand, there are concepts such as *Anzahlen,* while on the other hand, there are formal concepts, such as positive whole numbers, negative whole numbers, and so on. Although Husserl does not state it explicitly, what characterizes the first kind of concepts is clearly their material content or, in other words, the fact that they owe their meaning to concrete phenomena accessible on the empirical ground: this origin causes such concepts to be somewhat closed, to employ Husserl's expression, i.e. they cannot undergo any extension without turning into absurdities. Formal concepts, instead, gain their entire meaning from the definitions shaping them and from the rules prescribing how to manipulate them: since they do not owe their legitimacy to any concrete phenomenon, they can be widened and extended in order to suit the mathematicians' needs. In this regard, formal numbers are creations whose form depends solely on the purely analytic criterion of conforming to the law of contradiction: once this requirement is satisfied, no absurdity shall arise[52].

The subtle conceptual shift between *Anzahlen* and positive whole numbers was eased by the fact that the very same formal rules holding between the cardinals serve as a basis out of which the formal concepts of whole numbers are carved out; however, it is exactly this distinction that constitutes the focal point of Husserl's argument against the creative approach to the widening of the numerical field. As Stefania Centrone pointed out with great clarity, the reason for the author's dissent against Dedekind's strategy is to be found not so much in the method it adopts as in the concept taken as the starting point for the creative extension[53]. In fact, Husserl himself in his early writings on mathematics gives analogous accounts of how the extension of the numerical field should work:

51 *Ibidem*.

52 To investigate further on this topic see P. Spinicci, *recensione a* E. Husserl, *Studien zur Arithmetik und Geometrie*, ed. by I. Strohmeyer, «Rivista di Storia della Filosofia», 41, 1, 1986, pp. 177-187.

53 «Husserl's critique is not focused on a logical difficulty in Dedekind's theory […], but rather on a more philosophical problem: the formal procedures by which the expansion of the natural numerical field is obtained are correct, but Dedekind's *conceptual* presuppositions concerning the foundation of that expansion are not acceptable. The core of Husserl's argument is that *one cannot expand the concept of natural number* (*Anzahl*)» (S. Centrone, *Logic and Philosophy of Mathematics in the Early Husserl*, cit., p. 162).

the clearest example is contained in Chapter VIII of the *Philosophy of Arithmetic.* There, Husserl opposes Frege's proposal to assimilate zero and one to the *Anzahlen,* claiming that instead of quantifying an aggregate those notions deny its existence: replying "zero" or "one" to the question asking how many elements are in an aggregate implies that there is, respectively, no element or no multiplicity of elements at all. Once again, the material content of the concept of cardinal numbers sets us limits to their possible extensions: zero and one do not belong with them. However, Husserl immediately notes that it is reasonable to include zero and one among numbers if one considers the method through which they are obtained, namely that of a gradual addition of units: just as three is two units plus one unit, two units are one unit plus one unit and one unit is the addition of one unit where no unit was previously given. This constitutes a preliminary, embryonic, extension of the numerical field, and it is owed to purely operational reasons: zero and one are essential for configuring a proper numerical system through which more complex and more formal computing can be achieved. Hence, in agreement with Dedekind in this regard, Husserl too acknowledges that it is the operations that must lead the widening of the numerical field: the requirement of their encompassing executability leads to newly defined numbers. Instead, Dedekind's mistake consists in not differentiating between materially determined concepts, such as the *Anzahlen,* which are a closed manifold and begin no sooner than the cardinal two, and formal concepts, shaped by the definitions imposed on them according to operational requirements, such as the integers.

6. A Glimpse Ahead

Before concluding the present paper, I would like to show how meaningful the topics addressed in §§3-4 are for Husserl: to this end, I will look at §9 of his latest work, *The Crisis of European Sciences and Transcendental Phenomenology*. There – although from a significantly different and broader point of view, which I will not discuss here[54] – two of the themes discussed by Husserl in the *Philosophy of Arithmetic* reappear, namely A) the legitimacy of considering mathematical objects as primary contents of experience – or, in a non-phenomenological fashion, as existing in the external world – and B) the role of numerals in calculation.

A) §9 of *The Crisis of European Sciences and Transcendental Phenomenology* opens by stating that «for Platonism, the real had a more or less perfect methexis

[54] For more thorough readings of *The Crisis*, see D. Moran, *Husserl's* Crisis of the European Sciences and Transcendental Phenomenology. *An Introduction,* Cambridge University Press, Cambridge 2012; and P. Spinicci, *Il mondo della vita e il problema della certezza. Lezioni su Husserl e Wittgenstein*, CUEM, Milano 2000.

in the ideal»[55]. By «methexis» Husserl evokes the conception according to which the phenomena that populate our world are but the derivative outcome of a truer dimension, which would allegedly constitute the deepest level of being, namely its ideal component. The development of Husserl's philosophical reading of the history of modern thought revolves around the key role that the presupposition of methexis would have played: from the late sixteenth century on, the Platonic proposal of a realm beyond the sky would have been translated into that of a nature intrinsically codified according to mathematical language – as Husserl puts it, «through Galileo's *mathematization* [*Mathematisierung*] *of nature, nature itself* is idealized»[56]. Modern thinkers saw in mathematics the ideal component of reality, and moved it *within* experienced phenomena: according to this idealized conception of nature, mathematical objects were deemed to constitute the inner and truer core of what is ordinarily perceived. In Husserl's reading, in the life world, i.e. in the world as we know it before any attempt to offer a theoretical account of it, anything that manifests does so within a range of approximation or typicality. With an example, leaves from the same species of tree will never be exactly the same, but will nonetheless share a certain degree of similarity, which allows for distinctions between a certain type of leaves, belonging to a species of tree, and another type of leaves, belonging to another species. These *essential* fluctuations in the material objects of intuition, when considered all together, determine the habit of the life world, namely the fact that phenomena within it manifest, not without variations, according to a global style[57]. However, instead of elaborating a comparable descriptive account of the world and its phenomena, modern thinkers chose to provide an explanation for them: consequently, below the approximate style characterizing intuitable objects they posed the ideal of mathematical exactness, understood as the metaphysical cause underlying nature. It follows from this perspective that experience is split: on the one hand, there are phenomena as they enter the subject's field of perceptual experience, being thereby modified according to the functioning of human apparatuses; on the other hand, there are phenomena as they truly, ideally, are, that is, phenomena as they are understood and explained by mathematics and natural sciences – in other words, as the *res extensa* subject to motion.

55 E. Husserl, *Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie: eine Einleitung in die phänomenologische Philosophie*, ed. by W. Biemel, Martinus Nijhoff, Den Haag 1976, p. 20 (*K* from now on); Eng. trans. by D. Carr, *The Crisis of European Sciences and Transcendental Phenomenology. An Introduction to Phenomenological Philosophy*, Northwestern University Press, Evanston 1970, p. 23 (*C* from now on). On the notion of methexis in *The Crisis* see the above quoted P. Spinicci, *Il mondo della vita e il problema della certezza*, cit.

56 *C* p. 23; *K* p. 20.

57 Cf. subsection b) from §9 of *The Crisis* for the notions of «approximation», «typicality», «habit» and «style» of the «life world».

This is the angle from which in *The Crisis* Husserl assesses the modern theory of primary and secondary qualities: once again, he proves to be against the idea of a reality that is mathematical in itself, as Baumann – although for altogether different reasons – proposed. The modern conception of methexis is responsible for overturning an essential order of priority. Due to implanting ideal objects in the ground of perceptual experience, lived phenomena were ultimately downgraded from their primal role in the life world, since their mathematical components appeared as the only nucleus of truth in them. By appealing to the typicality of objects of intuition, phenomenological description reveals the falsity of such an account:

> In the intuitively given surrounding world […] we experience "bodies" [*Körper*] – not geometrical-ideal bodies but precisely *those* bodies that we actually experience, with the content which is the actual content of experience[58].

Neither numbers nor, more generally, mathematical concepts – such as pure extension – belong to a metaphysical structure shaping the external world, and in the life world «we find nothing of geometrical idealities, no geometrical space or mathematical time with all their shapes»[59]. The misunderstanding to which methexis leads is that of mistaking a method to explain reality for reality itself. With other words, the scientific modeling of the life world through mathematical techniques is not a neutral understanding of how the world truly is: its modeling constitutes an addition to the world, not its faithful analysis.

On the one hand, geometry stems from the open possibility of perfecting typical shapes that already exist in the life world and that contain in themselves inspirations for that perfecting: driven by the concrete needs of life, human practice recognizes in the actually perceived shapes, such as a wooden plank, hints towards more refined ones, such as an ideally smooth plane. The outcome of geometrical practice is the recognition of ideal objects: these are *pure* geometrical shapes, thought of as ideally infinite approximations starting from what is actually available to perception in the life world. On the other hand, mathematics proves a formidable language to translate – more or less directly – into «"functional" dependencies [*"funktionalen" Abhängigkeiten*] of numbers»[60] a multiplicity of data: idealized shapes and their geometrical relations, features qualifying objects from the life world, empirical interrelations between spatiotemporal facts. Thus, geometry and mathematics together allow for an exhaustive reading of the life world: the totality of its phenomena, once idealized, can be cast into a

58 *C* p. 25; *K* p. 22; italics in the original.
59 *C* p. 50; *K* p. 50.
60 *C* p. 41; *K* p. 40.

rigid net where their behavior and their nexuses are foreseen according to a comprehensive notion of causality whose rule is mastered thanks to mathematics. As accurate as this prediction may be, however, the point remains that mathematics and geometry remain tools to develop a possible reading of reality, and do not constitute the one true insight into it:

> Mathematics and mathematical science, as a garb of ideas [*Ideenkleid*] […], encompasses everything which, for scientists and the educated generally, *represents* [*vertritt*] the life-world, *dresses it up* [*verkleidet*] as "objectively actual and true" nature. It is through the garb of ideas that we take for *true being* what is actually a *method* – a method which is designed for the purpose of progressively improving, *in infinitum*, through "scientific" predictions, those rough predictions which are the only ones originally possible within the sphere of what is actually experienced and experienceable in the life world [*innerhalb des lebensweltlich wirklich Erfahrenen und Erfahrbaren*][61].

Stressing the purely methodical significance of mathematics and geometry is tantamount to highlighting how little their objects are already present as such in the life world or constitute its internal backbone. Natural sciences rest on the ingenious synergy between geometrical idealizations and mathematical relations: these are powerful lenses through which nature and the rules of our perception can be mastered, but they require to be fine-tuned and *applied* to the life world from the outside. On the contrary, believing that they carve phenomena from the inside, thereby positioning the ideal within the real according to the modern reception of methexis, leads to the metaphysical misunderstanding that urged Husserl to write *The Crisis*.

B) A major obstacle that during the Modern Age prevented a proper understanding of mathematics was the massive use of symbolic notations, the alphabet through which mathematical formulae (*Formeln*) were written. According to Husserl, formulae constitute the quintessence of Galileo's new physics, since they mirror the newly conceived ideal of an exact and all-encompassing causality capable of explaining any happening in the world. Pursuing this ideal, modern physicists took an interest in natural phenomena only insofar as each of these constituted an «example» (*Exempel*), a singular instance of a more general variable in the functional dependence expressed by a law and its respective formula. Thus, in the eyes of scientists the life world is replaced by a network of general numerical relations describing causal interactions:

> The indirect mathematization of the world […] gives rise to general numerical formulae [*Zahlformeln*] which, once they are formed, can

[61] *C* pp. 51-52; *K* p. 52.

> serve by way of application to accomplish the factual objectification of the particular cases to be subsumed under them. The formulae obviously express general causal interrelations, "laws of nature", laws of real dependencies in the form of the "functional" dependencies of numbers. Thus their true meaning does not lie in the pure interrelations between numbers (as if they were formulae in the purely arithmetical sense); it lies in what the Galilean idea of a universal physics […][62].

By virtue of their generality, formulae are taken to express «the true being of nature itself»[63].

As mathematical knowledge grows, one forgets that numbers are always values or measurements attributed to something, hence «*determined* [*bestimmten*] numbers», and deals freely with «*numbers in general* [*im Allgemeinen*], stated in general propositions»[64]. The price to pay for gaining the generality with which mathematical practice is familiar is the loss of intuitive content: formulae are general because the mathematical objects they evoke and employ – numbers in the first place – are untied from their original function of assigning a value to *things*. Husserl's reference here is the major change undergone by geometry since Descartes' contribution and the invention of calculus: with the development of analytic geometry, the discipline was subjected to an «arithmetization» (*Arithmetisierung*), its drawn figures were turned into numbers, its intuitive components were translated into algebraic equations. As a result, mathematical practice experienced its «technization» (*Technisierung*) and consequently superseded a reflective thought rooted in intuitions:

> In algebraic calculation, one lets the geometric signification recede [*zurücktreten*] into the background as a matter of course, indeed drops [*fallen*] it altogether; one calculates, remembering only at the end that the numbers signify magnitudes[65].

The shift from intuition to the manipulation of numbers led to the «emptying» (*Entleerung*) of the meaning of science which lies at the core of §9 of *The Crisis* and of its third appendix, and to which a crucial contribution comes from symbolic notation indeed.

Doing mathematics symbolically means complying uniquely with the signs involved in calculations, ignoring that what signs stand for – numbers in the first place – originally bore a more or less direct reference to the contents of

62 *C* p. 41; *K* p. 40.
63 *C* p. 44; *K* p. 43.
64 *Ibidem*.
65 *C* p. 44; *K* p. 44.

intuitions. However, doing mathematics symbolically does not necessarily imply calculating as a machine would: a symbolic activity does not prevent one from making relevant progress or discovering new truths within the mathematical realm. Rather, mathematicians working exclusively on symbols generally lack full awareness of the meaningfulness of their calculations. This is because symbolic notations and the formulae written in their alphabet open the way for mathematics to turn into the aforementioned technique:

> A mere art [*Kunst*] of achieving, through a calculating technique [*rechnerische Technik*] according to technical rules [*technischen Regeln*], results the genuine sense of whose truth can be attained only by concretely intuitive [*sachlich-einsichtigen*] thinking actually directed at the subject matter itself. But now (only) those modes of thought, those types of clarity which are indispensable for a technique as such, are in action[66].

Concretely intuitive thinking is precisely what modern mathematics and natural sciences renounced as they began to pursue an all-encompassing knowledge expressed through general laws and built upon formulae. From the symbols, such thinking looks back at (*einsehen*) things (*Sachen*) as they are, i.e. to the life world's phenomena as they are given to intuition, and by doing so it understands mathematics as a tool to foresee how those phenomena will behave. On the contrary, once a system of symbolic notation has been perfected, there is no reason why one should resort to intuited things: no "*Einsicht*" of them is required in order to calculate and obtain the needed results.

Letters and signs for relations, such as '+' and '×', allow for the construction of a calculating system through which accurate knowledge of phenomena can be gained without looking back at phenomena themselves – and here lies the dangerous enchantment held by signs on modern thinkers. This is why Husserl equates symbolic systems for computation and chess: to be fairly played, the game requires only that one abides by the rules prescribing how to move the pieces on the board; analogously, symbolic calculating requires only that one manipulates written signs following specific rules: no reflection on the meaning of those manipulations is needed. Indeed, *Philosophy of Arithmetic*'s ultimate result consisted of identifying arithmetic with purely symbolic computation, provided that any signs derivation could be backed up by actual theoretical reasonings based on the symbolized concepts. According to Husserl, the justification for calculating symbolically resides in this possibility: playing at the game of arithmetic is legitimate because – when well played – it proves to perfectly match the deductions actually developed on the corresponding symbolized concepts. As

[66] *C* p. 46; *K* p. 46.

seen in §5, in the work from 1891 Helmholtz's misconception of arithmetic as a mere game entirely based on symbols was targeted precisely because he failed to see that beyond numerals there lay number concepts. He overlooked the fact that it is only because of concepts' meaningfulness that conventional number signs can in turn *receive* their meaning. However, whereas Helmholtz's distorted view on the nature of numbers followed from his deliberate defense of a nominalist stance, modern thinkers' misconception of mathematics depended on the superficial acceptance of a metaphysical and anti-phenomenological doctrine, methexis, in light of which they structured their understanding of the world.

In Husserl's reading, the technization of mathematics, i.e. the fact that its practice turned into a mere symbolic computation, does not constitute a mistake *per se*: on the contrary, the process is «perfectly legitimate, indeed necessary»[67]. The focus of his analysis rather aims at showing how modern science gradually lost *awareness* of the meaning of its methods: «Here the *original* thinking that genuinely gives meaning to this technical process and truth to the correct results […] is excluded»[68]. The modern misunderstanding of science cut the bond linking the intuitions possible within the life world to scientific theories. Galileo and his successors hid the intuitive origin of scientific explanations obtained through idealities, instead of anchoring them to lived phenomena and their typicality. Rather than reconstructing the genesis of the scientific models of the life world starting from the latter as their nonnegotiable origin, they succumbed to the allure of symbols. As a result of such unquestioned attraction, they ignored the methodical value of mathematics: the metaphysical doctrine of methexis had to seem to them as the only plausible answer to justify the astonishing explanatory power shown by mathematical sciences. Ultimately, Husserl's warning in §9 of *The Crisis* is against the perpetuation of an «unquestioned tradition [*unbefragte Traditionalität*]»[69], namely against the blind reception of what has already been discovered and the uncritical acceptance of a previously developed method which is no longer examined – if it ever was – in its original meaning: when unquestioned, the symbolic language of mathematics prevents the aware mastery of scientific knowledge.

67 *C* p. 47; *K* p. 46.
68 *C* p. 46; *K* p. 46.
69 *C* p. 47; *K* p. 47.