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Epistemic Logic and the Problem of Epistemic Closure

Davide Quadrellaro[1]

Abstract. This paper argues that propositional modal logics based on Kripke-structures cannot be accepted by epistemologists as a minimal framework to describe propositional knowledge. In fact, many authors have raised doubts over the validity of the so-called *principle of epistemic closure*, which is always valid in normal modal logics. This paper examines how this principle might be criticized and discusses one possible way to obtain a modal logic where it does not hold, namely through the introduction of impossible worlds.

Keywords. Epistemology, Epistemic Logic, Epistemic Closure, Rantala Semantics, Logic of Knowledge, Impossible Worlds.

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Author. Davide Quadrellaro, davide.quadrellaro@gmail.com.

1 Introduction

The purpose of this article is to describe a minimal logic of knowledge which can be used by epistemologists with different philosophical orientations. A first way to proceed is describing a modal logic based on a Kripke-semantics, specifying how the accessibility relation should be restricted in order to represent knowledge. However, it is not difficult to prove that this standard formal epistemological analysis implies the validity of the principle of epistemic closure, namely of the fact that, if one both knows that p and that if p then q, then he/she also knows that q. This principle, however, has been object of criticism and objections by some epistemologists. Therefore, if we are looking for a general modal logical framework that can be used by philosophers with different orientations, we have to construct a formal system where the closure principle does not hold. An interesting way to proceed is working with the semantics which has been developed by logicians to account for the paradox of the logical omniscience. In fact, if we introduce the "impossible worlds" and we construct a Rantala-semantics based on them, we obtain a weaker logic where the closure principle does not hold.

In the first part of this article I present the modal logic **T**, which is generally considered the minimal formal system for the logic of knowledge. Firstly I introduce the syntax and the semantics of modal logic, secondly I characterize how the accessibility relation R_a has to be restricted in order to obtain the logic **T**. In the second part I prove that the principle of epistemic closure follows from **T** and I try to underline some critical aspects of it. In the third part I introduce an alternative logic for knowledge where the closure principle does not hold, namely a modal logic with impossible worlds and a Rantala-semantics. Finally, in the fourth part, I evaluate this proposal, trying to underline both upsides and downsides of it.

2 The standard logic of knowledge

A first way to give a formal account to epistemological concepts such as belief and knowledge is to adopt the language of modal logic. Even if the modal operators $\diamond$ and $\Box$ are usually read as possibility and necessity, we can also adopt an epistemic interpretation of them. On this alternative reading we will translate a logical formula like $\Box p$ not as "it is necessary that p" but rather as "it is known that p", "it is believed that p" or "it is certain that p". Following each of these interpretations we can formulate a different modal logic, in order to formalize the specific features of the considered epistemic operator. In what follows I will be interested exclusively in the former of these alternatives and I will focus my attention on the *logic of knowledge*.

Working with an epistemological interpretation of modal logic, it is worth specifying who is the subject of the knowledge we are speaking about. If we read $\Box p$ simply as "it is known that p", the meaning of this operator remains not clear enough. What does it mean, in fact, that something is known? Does it mean that someone knows it? Or does it mean that everyone knows it? Therefore, in order to be as clear as possible, we should adopt a more intuitive terminology and make explicit the fact that we are working with a *propositional notion of knowledge* and within a logic of *individual agents*. The box operator will be substituted by a K (for "knowledge"), followed by a letter that indicates who is the agent that knows the considered proposition. Modal formulas will look, thus, like $K_a p$ and $K_b p$ and they will be read as "the agent a knows that p" and "the agent b knows that p". In what follows, we will be interested in formal systems with only one agent, but it is important to keep in mind that we can introduce many K-operators, in order to map the knowledge of more than one subject[1].

Let us now move, after these introductory remarks, to give a precise definition of the *syntax* of the propositional modal logic for knowledge. We proceed extending the alphabet of classical propositional logic with a knowledge operator K_a.

Definition 2.1 (Alphabet of Propositional Modal Logic for Knowledge)**.** An alphabet for propositional modal logic for knowledge is defined as the union of the following disjointed sets:

- A denumerable set of *atomic propositional variables* $\mathcal{P} = \{p_0, p_1, ...\}$.
- The set of the *logical connectives* $C = \{\neg, \wedge, \rightarrow\}$.
- The set of the *knowledge operator* $O = \{K_a\}$.
- The set of *auxiliary symbols* $\mathcal{A} = \{(,)\}$.

Given the alphabet, it is possible to define inductively the set of the formulas of the logic of knowledge.

Definition 2.2 (Formulas of Propositional Modal Logic of Knowledge)**.** The formulas of the modal logic of knowledge are given by the following definition by induction:

1. If φ is an atomic propositional variable, then φ is a formula.
2. If φ is a formula, then also its negation $\neg\varphi$ is a formula.
3. If φ and χ are formulas, then also their conjunction $(\varphi \wedge \chi)$ is a formula.
4. If φ and χ are formulas, then also the conditional $(\varphi \rightarrow \chi)$ is a formula.

[1]For the introduction of multiple agents see both ´ family=Hendricks, familyi=H., given=Vincent, giveni=V., ´ family=Symons, familyi=S., given=John, giveni=J., (2015, pp. 9-11) and ´ family=Holliday, familyi=H., given=Wesley H., giveni=W. H., (forthcoming, pp. 5-7).

5. If φ is a formula, then also $K_a\varphi$ is a formula.

6. Nothing else is a formula.

The *semantics* of the logic of knowledge is provided by a Kripke-structure, which is the standard way to interpret modal languages.

Definition 2.3 (*Kripke-structures*). Given a propositional modal logic of knowledge, a *Kripke-structure* $\mathcal{M}$ is a triple $\langle W, R_a, V\rangle$, where:

1. W is a non-empty set. Intuitively, W is a set of "possible worlds" or "possible scenarios".

2. R_a is a binary relation over W, i.e. a subset of $W \times W$. Intuitively, we read vR_aw as "the possible world w is epistemically accessible from the possible world v by the agent a".

3. V is a function that assigns to every atomic propositional formula a subset of W. Intuitively, V specifies in which possible worlds each atomic formula is true.

Given the Kripke-structures, we can define the notion of truth in a world:

Definition 2.4 (*Truth in a world*). Given a propositional modal logic for knowledge, a Kripke-structure $\mathcal{M}$ and a world w, the notion $\mathcal{M} \vDash_w \varphi$ of being true in a world is defined as follows:

1. when φ is atomic, then $\mathcal{M} \vDash_w \varphi$ iff $w \in V(\varphi)$;

2. when φ has the form $\neg\chi$, then $\mathcal{M} \vDash_w \varphi$ iff $\mathcal{M} \nvDash_w \chi$;

3. when φ has the form $(\chi \wedge \psi)$, then $\mathcal{M} \vDash_w \varphi$ iff $\mathcal{M} \vDash_w \chi$ and $\mathcal{M} \vDash_w \psi$;

4. when φ has the form $(\chi \rightarrow \psi)$, then $\mathcal{M} \vDash_w \varphi$ iff $\mathcal{M} \nvDash_w \chi$ or $\mathcal{M} \vDash_w \psi$;

5. when φ has the form $K_a\chi$, then $\mathcal{M} \vDash_w \varphi$ iff for every possible world v such that wR_av, $\mathcal{M} \vDash_v \chi$.

The definition of truth in a world allows us to define two further important notions. We say that a formula φ is *true in a model* $\mathcal{M}$ if and only if it is true in every world $w \in W$ of the Kripke-structure $\mathcal{M}$. We say that a formula φ is a *valid formula* if and only if it is true in every world $w \in W$ of every Kripke-structure $\mathcal{M}$.

What we have described so far is the minimal system **K** of modal logic, with the only peculiarity that the informal reading that we have assumed for the modal operator is "the agent *a* knows that...". Nevertheless, it is clear that to obtain a logic of knowledge this is not enough. What one needs, rather, is to specify the formal properties that are typical of knowledge and to represent them in the logic. Putting specific restrictions over the accessibility relation R_a, it is possible to obtain many modal logics stronger than **K**, where more principles are valid formula. The problem is that it is not sufficiently clear which modal system photographs in the correct way the formal properties of knowledge. Since the purpose of this article is to examine which logic can be accepted by epistemologists with different philosophical orientations, we will extend **K** only with those principles which are generally taken for granted in the epistemological debate. Therefore, the only restriction that we want impose to our logical system is that it has to satisfy the following principle:

(T) $K_a\varphi \to \varphi$

What (T) says is that, if one knows a proposition, then this very same proposition must be true. This does not only follow from any analysis of knowledge as true belief plus something, but it also seems to be a valid minimal description of the meaning of knowledge. Indeed, if one says that he/she knows that p but it is not the case that p, it seems reasonable to conclude that he/she *does not* know that p, but rather only *believes* that p[2].

If we want that the principle (T) holds in the logical framework that we are considering, we have to put a restriction on the accessibility relation R_a. More precisely, as we prove with the following theorem, we have to restrict our attention to those Kripke-structures where the accessibility relation is reflexive. The modal logic that we obtain when we work only with reflexive accessibility relations is called **T**.

Theorem 2.1. *Given the language of propositional modal logic and its Kripke-structure $\mathcal{M} = \langle W, R_a, V\rangle$, the formula (T) $K_a\varphi \to \varphi$ is a valid formula if and only if the accessibility relation R_a is reflexive.*

Proof: Assuming that the accessibility relations R_a in $\mathcal{M}$ is reflexive, then given any possible world $w \in W$ we have that wR_aw. Therefore, since $\mathcal{M} \vDash_w K_a\varphi$ holds, then $\mathcal{M} \vDash_v \varphi$ holds in every world v such that v is accessible from w. But for reflexivity we have that w is accessible from itself and, therefore, that $\mathcal{M} \vDash_w \varphi$. *Vice versa*, assuming that $K_a\varphi \to \varphi$ is a valid formula then, for every Kripke-structure $\mathcal{M}$ and every world w in it $\mathcal{M} \vDash_w K_a\varphi \to \varphi$. Given the semantics of the conditional, this amounts to say that it is not the case that $\mathcal{M} \vDash_w K_a\varphi$ and $\mathcal{M} \nvDash_w \varphi$. But, if R_a was not reflexive, we could construct a Kripke-structure such as $\mathcal{N} = \langle W, R_a, V\rangle$, with $W = \{v, w\}$ and $R_a = \{\langle w, v\rangle\}$. In $\mathcal{N}$ we have that, if $v \in V(\varphi)$ but $w \notin V(\varphi)$, then $\mathcal{N} \vDash_w K_a\varphi$ but $\mathcal{N} \nvDash_w \varphi$, contradicting our claim that $K_a\varphi \to \varphi$ is a valid formula. Therefore, R_a must be reflexive. ■

[2] This aspect is famously stressed by ´ family=Wittgenstein, familyi=W., given=Ludwig, giveni=L., (1969).

3 The principle of epistemic closure and its problems

In the previous part of this article I have introduced the modal logic **T**, in order to represent some minimal formal properties of knowledge. Moving a step further, it is now possible to prove an interesting result, which says that the principle of epistemic closure is a valid formula in **T**. Firstly, let us clarify what we mean with the name of "principle of epistemic closure".

(CP) If an agent knows that φ and he/she knows that if φ then χ, then he/she also knows that χ.

It is straightforward to translate this thesis into the language of the logic of knowledge. We thus obtain the following formal version of the closure principle:

(FCP) $(K_a\varphi \wedge K_a(\varphi \rightarrow \chi)) \rightarrow K_a\chi$

We can now prove the following theorem:

Theorem 3.1. *Given the logic of knowledge* ***T****, the formal closure principle (FCP) is a valid formula.*

Proof: We reason for absurd. If (FCP) was not a valid formula, there would be a world w of a Kripke-structure $\mathcal{M} = \langle W, R_a, V\rangle$, where (FCP) does not hold. Given the semantics of the conditional, this means that $\mathcal{M} \vDash_w K_a\varphi \wedge K_a(\varphi \rightarrow \chi)$ but $\mathcal{M} \nvDash_w K_a\chi$. Given $\mathcal{M} \vDash_w K_a\varphi$, we have that in every world accessible from w, φ holds. Given $\mathcal{M} \vDash_w K_a(\varphi \rightarrow \chi)$, we have that in every world accessible from w, $\varphi \rightarrow \chi$ holds. Moreover, since $\mathcal{M} \nvDash_w K_a\chi$, there is at least one world v such that wR_av where $\mathcal{M} \nvDash_v \chi$. But in this same world v we have that $\mathcal{M} \vDash_v \varphi$ and $\mathcal{M} \vDash_v \varphi \rightarrow \chi$ hold too, from which it follows that $\mathcal{M} \vDash_v \chi$. Therefore, we obtain the contradiction that $\mathcal{M} \vDash_v \chi$ and $\mathcal{M} \nvDash_v \chi$. ■

If our concerns are mainly epistemological this result has a particular relevance. In fact, what we have proved is that even if we work with a weak modal system, the principle of epistemic closure will hold in it[3]. Therefore, if we have some reason to refuse the principle of epistemic closure, then we can not adopt the formal logic **T** anymore, for it describes knowledge in a way which is inconsistent with our theory. In particular Dretske (1970) offers at least two possible reasons to refuse the closure principle[4]. In the rest of this part I will present both

[3]Notice, moreover, that in the proof of the theorem 3.1. we did not make any use of the fact that the accessibility relation between worlds is reflexive. Therefore, our proof is valid also for the basic modal logic **K**.

[4]´ family=Luper, familyi=L., given=Steven, giveni=S., (2016) synthesizes a wide range of arguments against the closure principle, often originally raised by Dretske and Nozick. However, even if Luper's reconstruction is clear, I do not agree with his presentation of the arguments from the "analysis of knowledge". In fact, the theories of knowledge supported by Dretske and Nozick are *explanations* of why the closure principle fails and not *reasons* to refuse it. Luper commits, therefore, a sort of inversion of the right order of explanation.

of them, but I will not try to set the question about their validity. Indeed, I only want to show that it might be reasonable for an epistemologist to reject the closure principle. In fact, given the possibility that (FCP) is not acceptable, we have to look for a modal logic for knowledge weaker than the standard one described by the Kripke-structures. Our purpose, in fact, is not to take part in the epistemological debate and to identify the modal logic which better describes knowledge but, rather, it is to find a minimal logical framework which can be accepted by epistemologists of different currents.

A first critique to the principle of epistemic closure is linked to skepticism. In fact, one general way to reconstruct the argument presented by the skeptic is with the following argument:

(1) I do not know that I am not a brain in a vat
(2) If I do not know that I am not a brain in a vat, then I do not know that I have hands.

---

(3) I do not know that I have hands $\therefore$

The premiss (2) of this argument is a consequence of an instance of (CP). If I know that I have hands and I know that if I have hands I am not a brain in a vat, then I know that I am not a brain in a vat. Therefore, if I do not know that I am not a brain in a vat, then either I do not know that I have hands, or I do not know that if I have hands I am not a brain in a vat. However, since I know that if I have hands I am not a brain in a vat, we can exclude the second disjunct and obtain (2): if I do not know that I am not a brain in a vat, then I do not know that I have hands[5].

If skepticism is expressed in the form of the syllogism presented above, there are two main strategies to criticize it. Either one denies the premiss (1), either one denies the premiss (2), namely the closure principle. The first horn was chosen by Moore (1939), who reversed the skeptic's argument in its contraposed version[6].

[5]It is worth underlining that, in order to obtain (2) from (CP), we have to take for granted that we know that if we have hands we are not a brain in a vat. Although this might seem trivial, there are two problematic aspects which deserve some further reflections. On the one hand, one may think that it is much more reasonable to deny the premiss of the argument from (CP) to (2), namely to assert that we do not know that if we have hands then we are not a brain in a vat, rather than to accept the conclusion it leads to, i.e. that we do not know that we have hands. On the other hand, there might be a skeptical scenario that we do not know, or a person who never thought about brains in a vat. But if one has never thought about a skeptical scenario, it does not seem plausible to say that he/she knows that if he/she has hands, then he/she is not in the considered skeptical scenario.

[6]For historical's sake, let me remark that Moore did not deal with the brain in a vat hypothesis in his original article of 1939, but he rather considered more traditional skeptical scenarios.

(1) I do know that I have hands

(2) If I do not know that I am not a brain in a vat, then I do not know that I have hands.

---

(3) I do know that I am not a brain a vat $\therefore$

However, this solution implies that we do actually know that we are not brains in a vat, which is a conclusion that many might find excessively strong. Therefore, if we want to remain faithful both to the intuition that we do know that we have hands, both to the intuition that we do not know that we are not brains in a vat, we have to abandon the closure principle. Notice that this is not an argument against skepticism. If we want to criticize skepticism *because* the closure principle does not hold we need independent arguments against (CP). On the contrary, this is an argument against the closure principle, *because* skepticism does not hold. So, what this argument needs are independent reasons to refuse skepticism.

However, Dretske criticizes the principle of epistemic closure also in a second more explicit way, bringing some counterexamples to it. Perhaps the most famous one is the so-called "zebra case". Imagine that you are in a zoo with your nephew. While you are walking around, he asks you if you know what is the animal you are looking at. You observe it, you notice that it looks exactly how you expect a zebra should look like, and you also find a sign with "zebra" written on it. Without any further doubt you would reply to your nephew's question something like: "Sure! It is a zebra". Thus, you do know that the animal you are observing is a zebra. But do you know that it is not a disguised mule? Indeed, it might be a mule so well depicted by the zoo-officers to look exactly like a zebra, maybe in order to attract more visitors.

Examples like this present a sort of strange situation. On the one hand, we have a plenty of reasons to believe that the animal we are observing is a zebra. On the other hand, we do not know that it is not a disguised mule. Moreover, we are also completely aware that mules and zebras are different animals. Therefore:

(i) we know that the animal we are looking at is a zebra;

(ii) we know that if the animal we are looking at is a zebra, then it is not a disguised mule;

(iii) we do not know that the animal we are looking at is not a disguised mule.

Clearly, (i), (ii) and (iii) taken together are an instance of failure of the closure principle.

Together, these two arguments show that the principle of epistemic closure is not so obvious and trivial as one might believe at first sight. A closer examination of it shows both that it has skeptical consequences and that it does not always fit our intuitions in concrete examples. Therefore, if we want to find a propositional modal logic which describes some minimal properties of knowledge generally accepted by epistemologists we have to weaken in some way the logic of knowledge that we have previously presented.

4 The impossible worlds and the Rantala-semantics

In the context of the logical literature, an alternative to the standard Kripke-semantics has been provided in order to account for the problem of logical omniscience. In fact, one further consequence of adopting a modal logic like **K** or stronger is that any agent knows every classical tautology. In fact, since classical tautologies are valid in every possible world, the agent always knows them, for they are trivially true in all the worlds which the agent has access to. Although it is important to keep distinct the problem of the epistemological closure principle from the one of the logical omniscience, we can try to apply the logical system used to answer to the latter of these problems also to respond to the former one[7].

Given the syntax of modal logic that we have already defined, we can introduce a slightly different semantics, namely a Rantala–semantics[8].

Definition 4.1 (*Rantala-structures*)**.** Given a propositional modal logic of knowledge, a *Rantala-structure* $\mathcal{R}$ is a quadruple $\langle W, W', R_a, V\rangle$, where:

1. W is a non-empty set. Intuitively, W is a set of "possible worlds" or "possible scenarios".

2. W' is a set. Intuitively, W' is a set of "impossible worlds" or "impossible scenarios".

3. R_a is a binary relation over $W \cup W'$, i.e. a subset of $(W \cup W') \times (W \cup W')$. Intuitively, we read vR_aw as "the possible or impossible world w is epistemically accessible from the possible or impossible world v by the agent a".

4. V is a function that assigns to every atomic propositional formula a subset of $W \cup W'$ and to every formula a subset of W'. Intuitively, V specifies in which possible or impossible worlds each atomic formula is true, and in which impossible worlds each formula is true.

As one can immediately notice, the difference between the Kripke and the Rantala structures relies on the introduction of a set of impossible worlds. To see how

[7]On the difference between the problem of logical omniscience and the one of epistemic closure see ´ family=Holliday, familyi=H., given=Wesley H., giveni=W. H., (forthcoming, pp. 8-10).

[8]The name of Rantala-semantics comes from the Finnish logician Veikko Rantala. Here I follow the presentation of its semantics given by ´ family=Wansing, familyi=W., given=Heinrich, giveni=H., (1990), who also provides an interesting comparison between the Rantala-semantics and other methods to solve the paradox of logical omniscience.

they affect the interpretation of every formula, we shall reformulate also the notion of truth in a model.

Definition 4.2 (*Truth in a world*)**.** Given a propositional modal logic for knowledge, a Rantala-Structure $\mathcal{R}$ and a world w, the notion $\mathcal{R} \vDash_w \varphi$ of being true in a world is defined as follows:

1. If $w \in W'$, namely if w is an impossible world, then $\mathcal{R} \vDash_w \varphi$ iff $w \in V(\varphi)$;
2. If $w \in W$, namely if w is a possible world, then:
   (a) when φ is atomic, then $\mathcal{R} \vDash_w \varphi$ iff $w \in V(\varphi)$;
   (b) when φ has the form $\neg\chi$, then $\mathcal{R} \vDash_w \varphi$ iff $\mathcal{R} \nvDash_w \chi$;
   (c) when φ has the form $(\chi \wedge \psi)$, then $\mathcal{R} \vDash_w \varphi$ iff $\mathcal{R} \vDash_w \chi$ and $\mathcal{R} \vDash_w \psi$;
   (d) when φ has the form $(\chi \rightarrow \psi)$, then $\mathcal{R} \vDash_w \varphi$ iff $\mathcal{R} \nvDash_w \chi$ or $\mathcal{R} \vDash_w \psi$;
   (e) when φ has the form $K_a\chi$, then $\mathcal{R} \vDash_w \varphi$ iff for every possible or impossible world v such that wR_av, $\mathcal{R} \vDash_v \chi$.

It is now possible to clarify which is the role that the impossible worlds play in the new structure now defined. A first notable aspect is that, while in regards of the possible worlds the notion of truth in a world is defined inductively, the truth-value of every formula in an impossible world is directly specified by the assignment V. In an impossible world we might have that a disjunction is true even if its two disjuncts are both false, or that even if two formulas are true their conjunction is false, and so on. The distinguished aspect of this structure is that the anomalous behaviour of impossible worlds has some consequences on the evaluation of formulas in “normal” possible worlds. In fact, in order for a modal formula like K_ap to be true in a possible world w, the formula p has to be true in every world v, both possible and impossible, such that wR_av.

The notion of valid formula has now to be defined for the new Rantala-semantics: we say that a formula φ is a *valid formula* if and only if it is true in every possible world of every Rantala-structure. Given this new definition and thanks to the introduction of the impossible worlds, we can show that the principle of epistemic closure (FCP) is not a valid formula anymore. In fact, even if $\mathcal{R} \vDash_w K_a\varphi$ and $\mathcal{R} \vDash_w K_a(\varphi \rightarrow \chi)$, it is still possible that $\mathcal{R} \nvDash_w K_a\chi$, since there might be an impossible world i such that wR_ai where $i \in V(\varphi)$ and $i \in V(\varphi \rightarrow \chi)$ but $i \notin V(\chi)$.

Moreover, notice that the introduction of impossible worlds does not imply that “everything goes”. We can, as we have already done for **K**, propose a strengthening of this logical framework in order to meet at least the essential properties of the knowledge operator. Exactly as we have argued in the first part of this article, the minimal requirement for a logic of knowledge seems to be that if we know a proposition, then this very proposition is true. Again, if we impose that the accessibility relation is reflexive, then we obtain a logic where the formula (T) $K_a\varphi \rightarrow \varphi$ is a valid formula. In this way we can define the new logic **T'**, obtained by considering only those Rantala-structures where the accessibility relation between worlds is reflexive.

5 An evaluation of the Rantala-semantics strategy

In this last part I shall draw some consequences from the previous analysis and try to evaluate if the Rantala-semantics that we have defined provides a minimal logical framework to describe the formal properties of knowledge. Firstly, I argue that it is possible to identify two reasons to believe that the Rantala-semantics actually describes a valid minimal logic of knowledge. Then I will consider two objections. While one will result to be only an apparent critique to the Rantala-semantics strategy, the second one will identify a true limit of it.

(i) A first observation is that the logic **T'** that we have defined actually provides the minimal logical framework for knowledge which we were looking for. On the one hand, the principle (T) $K_a\varphi \rightarrow \varphi$ results to be a valid formula in this system: working in **T'** we can represent the fact that if an agent knows a proposition, then that proposition is true. On the other hand, the logic **T'** does not force us to accept the closure principle, since (FCP) is not a valid formula in it. Therefore, epistemologists with different theories about knowledge can all accept the modal system **T'** as a minimal framework, which reflects only those properties of knowledge which are unanimously recognized.

(ii) Moreover, the Rantala-semantics is sufficiently flexible to provide not only a minimal common framework, but also a basis suitable for further developments. Given the minimal logic **T'**, it is possible to obtain systems with new axioms or inference rules imposing new conditions on the accessibility relation R_a or on the evaluation function V[9]. In this way, the Rantala-semantics can be used also to represent more complex theories of knowledge, in which more principles hold and should be treated as valid formulas. Epistemologists of different philosophical orientations will thus share the common framework given by **T'**, and they will also be able to describe more complex and rich systems without the need of describing a new and different semantics. Even if **T'** is a quite general and minimal system, we can start from it and obtain step by step new and stronger logics, which will formalize richer and more complex accounts of knowledge.

(iii) However, one aspect of the Rantala-semantics that some philosophers may find problematic is the fact that it makes use of impossible worlds. In fact, even if we accept to work with the framework of possible worlds of the Kripke-

[9] Compare with ´ family=Wansing, familyi=W., given=Heinrich, giveni=H., (1990), who also presents some examples of restriction.

structures, the introduction of impossible worlds poses some new problems. Indeed, although possible worlds represent sets and combinations of facts and events that are not actual, they are still consistent with the laws of classical logic. Differently, it is not straightforward to account for worlds where the most evident logical contradictions may hold. In an impossible world both a proposition and its negation might be true, two disjuncts can be true and the entire disjunction false, and so on. Nevertheless, even if impossible worlds surely present paradoxical features, I think that this problem is only apparent.

Firstly, as Nolan (2013, p. 367–370) underlines, almost every metaphysical theory about the possible worlds can be extended in order to account also for the impossible ones. The only theory which has some problems while explaining the nature of impossible worlds is modal realism, which regards possible worlds as entities really existing. However, there are also some attempts to extend the modal realist perspective in order to describe impossible worlds[10]. Moreover, one may also decide to follow an alternative direction and to consider the useful theoretical role of the impossible worlds a valid reason to reject modal realism and to defend another metaphysical perspective also in regards of the "normal" possible worlds.

Furthermore, it is not obvious at all that the introduction of impossible worlds in epistemic logic forces us to take an explicit position about their metaphysical nature[11]. In fact, the specific philosophical problems that a modal logic raises are linked to the informal interpretation that we decide to give of its operators. For instance, if we read the box symbol as representing necessity, then we have to clarify what does it mean that a proposition is necessary in a world w if and only if it is true in every possible world which is accessible from w. An analysis of the nature of possible world is essential, in this case, in order to make sense of the metaphysical interpretation of the system of modal logic that we are considering. However, if the reading that we are adopting is epistemic, we do not need to take such a metaphysical attitude. As we have already said defining the Kripke-structures, the label of possible world can be substituted without any problem with the one of "scenario". Indeed, the possible and impossible worlds are only the combinations of facts and events that an agent may find plausible descriptions of the reality or not. The informal epistemological reading of the knowledge operator does not call for any metaphysical interpretation. The fact that an agent knows a proposition if and only if that proposition is true in every world to which he/she has access only means that that proposition is part of all the descriptions that he/she considers as possibly valid representations of the reality.

[10] Compare with ´ family=Nolan, familyi=N., given=Daniel P., giveni=D. P., (2013, p. 369).

[11] ´ family=Wansing, familyi=W., given=Heinrich, giveni=H., (1990, p. 536) takes an even stronger position, saying that the question itself about the nature of the impossible worlds is "unsatisfactory".

(iv) Ultimately, despite its many virtues, I think that it is possible to identify a proper limit of the Rantala-semantics strategy. Let us distinguish two different aspects: the failure of the closure principle itself and the explanation of the fact that it does not hold. Depending on what we ask to an epistemic logic, we might then give different evaluations to the Rantala-semantics strategy. On the one hand, as I have already pointed out, the modal logic **T'** offers a formal system where the closure principle of knowledge is not a valid formula. If we adopt **T'**, indeed, we are able to represent many formal properties of knowledge and to potentially adjust the system – working on the accessibility relation and the evaluation function – to meet the characteristics of different epistemological theories. On the other hand, the Rantala-semantics does not provide an explanation of why the closure principle fails. Or, even worse, one may argue that it actually gives a *wrong* explanation of this fact. Indeed, the "cause" that determines the failure of (FCP) in the Rantala-semantics is the introduction of the impossible worlds. If we try to interpret this formal aspect from an epistemological perspective, the result is that the epistemic closure principle does not hold because the agent consider as plausible descriptions of the reality also scenarios where the laws of logic do not hold. However, the problem is that this is not the explanation that the epistemologists who refuse closure – notably Dretske and Nozick – have provided. Therefore, even if it offers a framework that can be accepted also by the epistemologists who do not accept the closure principle, the Rantala-semantics do not reflect in any way their intuitions about why this principle does not hold[12].

Finally, trying to sum up the considerations developed in this last part, it is possible to sketch an evaluation of the Rantala–semantics strategy. The result that we obtained can be regarded as twofold and it depends on what we ask to an epistemic logic. If we want a strong characterisation of a formal system, such that it reflects all the theoretical features of an epistemological theory, then the Rantala–semantics strategy does not seem to be the right way to account for the problems presented by the closure principle. Still, a more modest attitude is also possible. In fact, we can demand to a formal system only to verify as valid those principles – and only those – which an epistemological theory regards as the formal properties of knowledge. In this light, even if it does not provide any heuristic insight about the failure of (FCP), the Rantala-semantics is an interesting common framework for different epistemological perspectives, which can also be refined and strengthened in further ways.

[12]An interesting contribution on this topic is ´ family=Holliday, familyi=H., given=Wesley H., giveni=W. H., (2015), who directly formalizes the epistemological theories proposed by Dretske and Nozick. Notice, however, that although in this way a formal system gains in heuristic power, it also loses the generality that makes it acceptable by epistemologists with different ideas.

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