



Problems with definition of relational entities within spacetime And a possible solution

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Abstract. The contemporary debate on definition of relational entities existing within spacetime is mostly characterized by the belief that a multiplicity of entities of such kind exists. The aim of this paper is to show how this statement leads to some issues of diverse nature and trying to propose instead a metaphysical position that might do without it.

First I'll show why we should consider the relevant entities as relational. Nevertheless I'll show that my argumentation can also run against entities primitively individuated by monadic properties. Then, taking into account different theories, (stage theory and worm theory in particular) I will show how special relativity bring to some issues. The positions following those issues lead to problems that are, in my opinion, insurmountable for what concerns the causal relation. Moreover, even pretending we can solve those problems, there is a counterexample to stage theory and worm theory in Minkowski spacetime by cases in which symmetry between relational entities force us to admit multilocation in order not to loose the identity principle of indiscernibles. Eventually I will analyse a counterexample from quantum mechanics to the identity principle of indiscernible. Given these distinct arguments, I will outline the borders of a metaphysical monist position with the purpose of showing that it is a solution to each, compatible with both the special relativity and with quantum mechanics. This is a existence monism in which we talk about several entities by selecting sections of the only existing object with properties we choose. In this view there would be no need for Identity Principle for Indiscernibles because it lacks any multiplicity

of entities to define apart. Moreover it would be a determinist position without the need of any causal relation. In the same way in which the right part of a don't cause its left part we could say that the temporal part t doesn't cause the temporal part t_1 of the universe.

Keywords. Relational Entities, Minkowski Spacetime, Proper Time, Monism.



Introduction

One may say that there are no reason to define concrete entities as relational assuming those as primitively individuated. In this position two H₂O particles *a* and *b* with all the monadic properties in common would be defined apart by the fact that those are primitively individuated in different places. Now, given that we don't want this primitive property to be relational we would say that *a* is not *b* because there is no cases in which they have all monadic properties in common they share every monadic property except being located at x_a which is only a property of *a*. The problem with this position is that said properties wouldn't give any partition of the domain being referred exclusively to each entity which guarantees the identity. It would be a theory totally in lack of explicative power defining *a* apart from *b* in virtue of a property which apply exclusively to *a* and to nothing else in the universe. It would be like saying that *a* is not *b* because *a* is *a*, in fact the only reason why they're defined apart is a property that can in principle apply exclusively to *a*. Normally we look for more explicative position, otherwise, let's say: the personal identity problem could be solved saying that Daniele stay identical to itself trough time because keeps having the primitive property (the only relevant) of being Daniele which apply exclusively to it. Basically all the identity problems could be solved just applying this method and defining each entity we want identical to itself on a primitive property which apply exclusively to it. From now I'll assume we don't want such redundant position which, though being in safe from a multitude of objections would be not explicative at all, not giving actually any account for definition of entities existing within spacetime. So I'll assume we want relational properties as relevant for the definition because we need to define apart two H₂O particles with all the monadic properties in common.

I'll consider the questions pertaining the persistence of relational entities looking for a definition of entities possessing relational properties within spacetime. It doesn't seem difficult to establish questions relating to the identity of the entity with themselves when their relational properties change, to problems related to the definition that we want to give them.

Distinguishing, as suggested by Sider, three-dimentionalism and four-dimentionalism, we can say that, for three-dimentionalism, to ensure the identity of *a* over time it's sufficient to ensure the identity of type. *a* at time t_0 is the same entity at time t_n because it retains its kind while other properties have changed. Unfortunately, if the only thing relevant for the definition of an entity is its type, then all the entities of the same type are the same entity. Conversely, if the relational properties are relevant to distinguish entities of the same type, then we are introducing them in the definition of the entity and we're falling back into Stage Theory. About four-dimentionalism: Balashov, (2002) takes in

exam two quadridimensional theories – Worm Theory and Stage Theory – with the aim of showing that both agree with the results of special relativity. My purpose is to demonstrate this conclusion as wrong because of problems with the notion of proper time, and with multilocation. Moreover I'd like to jump in a debate between Calosi, (2015), Davidson, (2014) and Balashov, (2014) about extrinsicity of four-dimensional shapes. Calosi's position suggest that it is possible to apply Lotentz's invariance for the definition of existing relational properties within spacetime. My aim is to show that both Stage theory and Worm theory encounter problems in a position of this kind.

1 Stage Theory

Stage Theory, in order not to compromise itself with the idea that all entities that share every monadic properties are the same entity, must admit that the relational properties of instantaneous entities are relevant for the definition. This description seems to encounter some difficulties when it meets the description of special relativity. Taking into account any a entity and defining time in terms of relational properties, we will say that, in the S_0 reference system, a is located at temporal distance Δt_0 from any object b in S_0 , while from S_1 the "same" a is located in the time interval Δt_1 from the "same" object b observed from S_1 (given Stage Theory, of course it becomes improper to say "the same object b ". We should say "the object b_2 treated as it were b_1 "). In the case of dilation of times, we will have separate entities depending on the reference system from which they are observed. To prevent this, we could exploit the special relativity's notion of spacetime distance.

In Minkowski spacetime, it is possible to determine spacetime-invariant distances between two points according to the equation $\Delta d^2 = c^2 \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ where t is the measure of time in a frame of reference whatsoever, and x, y, z are the spatial coordinates, in the same reference system.

So, having an invariant distance, we could define relational entities within spacetime considering it as relevant for the definition of them. Then we could say, in the aforementioned case, that it can be described by saying that S_0 integral with a observes a_2 (defined on proper time of a) in what t_2 is for it (defined on the proper time of S_0) and that S_1 in relative motion observes a_5 (defined on the proper time of a) in what t_2 is for it (defined on the proper time of S_2).

All distances between entities placed so would be defined in an invariant manner, but unfortunately at the cost of introducing a quite complex notion as that of proper time. It, defined as the time measured in a inertial reference system integral with the phenomenon of which we measure duration, is based on a naive idea of persistence of concrete entities in time. If we are to assume a naive persistence of a in time to define its proper time and so each of the enti-

ties $\{a_1, a_2 \dots a_n\}$ ranging from t_1 to t_n proper for a (or S_0), then we are using the definiendum in definiens. Strictly speaking, moreover, in stage theory nothing exists where common sense and physics see a .

So S_0 in what for it is the time t_2 observes a_2 while S_1 in what for it is the time t_2 observes a_5 that S_0 will see only when it will get to what for it is the time t_5 . We can define the two a_2 and a_5 entities based on their spatiotemporal distances, invariant in Minkowski spacetime, via any asymmetric event. It remains to be determined in what sense a_5 is being measured by S_0 in what for S_0 is t_5 whereas it is being measured by S_1 in what for S_1 is t_2 . One might imagine that there is no need to account for this but not doing so would make the introduction of spacetime distance useless. We opted for this option to give an account of the phenomenon of times dilation; renouncing to do so would cast doubt on the need to use spacetime distance as criteria to define the entities. Physics encounters no difficulties in introducing proper time and a trivial persistence of entities. Unfortunately, philosophy is no easy game. Copying the description of physics to solve problems is not enough. In the present case, doing so would involve the use of the definiendum in the definiens, as shown above.

The whole notion of inertial frame of reference is based on the persistence of concrete entities at the changing of their relational properties. Persistence which of course is not simply given. Any time axis in Minkowski spacetime is something's proper time and is defined in these terms.

We could say that being integral with a is being integral with each of the instantaneous entities which compose it. Having so defined a and its being integral to S_0 , we would overcome problems with its proper time. We define an inertial reference system as a system where if a material point is free, meaning not bending to forces or bending to null resultant of forces, then it will preserve his state of rest or uniform motion until it will be perturbed (to put it more simply: the first law of dynamics is valid). In other words, any material point within the reference system measures no acceleration.

In both forms the notion of inertial reference system requires the presence of entities at the variation of their properties. Instantaneous entities do not conform to this description: they, for their existing in $\Delta t = 0$, don't preserve any state of rest or uniform motion. In $\Delta t = 0$ no entity measures acceleration. Note that Δt is something which is still possible to talk about in the new paradigm that we have introduced. With $\Delta x^2 = 0$; $\Delta y^2 = 0$; $\Delta z^2 = 0$ we have $\Delta d^2 = c^2 \Delta t^2$.

We could say that as some relational properties "change", $\{a_1, a_2, a_3 \dots a_n\}$ retain some others. Each element in the sequence of a integral with S_0 keeps the same distance from some k entity while their distance from some b entity in S_1 keep changing. Obviously we cannot conceive a as a sequence of instantaneous entities without doing the same with the rest. We shall define k as the sequence of entities $\{k_1, k_2, k_3 \dots k_n\}$ and b as the sequence of entities $\{b_1, b_2, b_3 \dots b_n\}$.

We would say that a_2 has the same distance from k_2 which a_1 had from k_1 while it has a different distance from b_2 with respect to the distance that a_1 had from b_1 . Thus a_2 is in S_0 in the sense that it has the same distance from k_2 that had a_1 from k_1 and is not in S_1 in the sense that it doesn't have the same distance from b_2 that had a_1 from b_1 . In Minkowski spacetime the set of points equidistant from a given point is a hyperbola, not a circle. We'd be then compromising ourselves with the idea that a can be integral to a frame of reference light years away from his proper time, moving away. To avoid this we could add a clause for which the only relevant distances for the definition of the reference systems are space-like. This definition, which obviously can not make use of proper time notion, requires that you can locate the next entity for any entity, then for each entity of common sense we have \aleph_0 maximum entities. Otherwise it wouldn't be possible to identify from time to time the entity a_n to which a_{n+1} must remain equidistant to stay in the reference system. We would be in a theory that consider quantization of spacetime in order to make possible applying the successor function to entities, which accords well with the latest physical theories about the nature of spacetime.

Unfortunately the position thus defined is susceptible of a counterexample: imagine a planet that orbits around a star following a circular orbit of uniform motion. Then, with a good approximation, the system can be defined as an inertial system (i.e. centripetal acceleration can be ignored). So we will have three objects: the planet g , the star q and a on the planet. At each time the distance between any g_n and a_n is the same distance that separated g_{n-1} and a_{n-1} but also the distance that separates g_n from q_n and a_n from q_n would remain the same. The vector spacelike which defines the distance between any g and corresponding a and q in the mental experiment will always have the same value because the planet describes a circle around the star. In the description just introduced the only one available reference system would be a system in which the three entities are not in relative motion, which is unacceptable. In fact there must be also a frame in which a and g are moving with respect to q .

The only way out seems to say that, looking at a from different frames of reference, various relational properties are observed and therefore different entities, according to Stage theory assumptions. Where before we had an only a (instantaneous) we will have $\{a_1, a_2, a_3 \dots\}$ as many as the possible reference systems. Then we will say that physics postulates an identity between an entity observed in S_0 and one observed in S_1 but, strictly speaking, there is no identity.

2 Bundle theory and Theory of tropes

Bundle theory is the theory according to which entities are nothing over and above the properties they instantiate. That is to say that where common sense

sees an ordinary macroscopic object really there is only a bundle of properties. Now: Bundle theory requires that relational properties appear in bundles. Attributing to bundles a position in spacetime that is relevant for their definition is in fact essential to avoid, for example, that our world would be populated by a single copy of each elementary particle of physics multilocated. We wouldn't say that the bundle a has the property of being at a certain distance from the bundle b . Both because we would be treating bundles in terms of substances to which properties can be attributed, and because talking about bundles within the definition of a property would implicate difficulties, of which I will speak afterwards talking about tropes. Even admitting that in the bundle theory things do exist as places does not solve the problem. "Being in the place xyz " can only be defined in terms of specific distances between entities or between places.

We should therefore say that relational properties are copresent with the others in any bundle a . Considering again the two bundles a and b , we would say that copresent to the monadic properties of a there will be the properties of being at a certain distance from each of the properties that compose b . This formulation would enable us to define the relational properties satisfactorily; unfortunately, however, it leads to an infinite regress. In the bundle a the properties of being at a certain distance from each of the properties that compose b will appear. So also in the bundle b there will be the properties of being at a certain distance from each of the properties that compose a . Then, if in a appears the property to be "at 10m from the property of being b " in b will appear the property of "being at 10m from the property to be 10m from the property of being b ". Note that I take here distance between properties as invariant distance between spacetime points at which those properties are instantiated. Of course such an acceptance allows also to define distances between sets of collocated properties.

Maybe it is possible to block the regress by claiming that in the bundle relational properties appear only with reference to non-relational properties. This, however, involves two separate problems: i) The distinction between relational properties and monadic ones may not have higher value than the explanatory one. It is not entirely clear how to draw a clear line between these two sets or if this is possible at all. ii) even if the distinction were perfectly clear, it would still be unclear why relational properties should not appear with reference to relational properties. This would seem to be an *ad hoc* solution to solve the recourse.

Whether the bundle theory decides to accept this regress, whether it decides to solve it by introducing a distinction between monadic and relational properties and an *ad hoc* hypothesis, this position would appear as a version of Stage Theory in the field of special relativity. In fact, the whole discussion regarding the Stage theory problems with relativity could be transcribed here. Since the

bundle theory considers the bundle's change (to which ontological value is not attributed), at the change of each property that composes it, the conclusions will be the same, albeit with a drastically different metaphysical assumption.

One might think that the argument used for the definition offered by the bundle theory repeats itself unchanged for the theory of tropes. Such a theory instead of reducing objects to bundles of universals take them to be bundles of particulars i.e. the redness of a certain door is numerically distinct from the redness, of the very same shade, of another door. Now: In order to define the relational entities in spacetime as bundles of particulars, is required to define the particulars involved. It seems necessary to clarify whether denoting by sign a a trope, the sign b will denote the same trope or another. It is not clear how else we might put the green of a and the green, exactly the same shade, of b as particular separate entities and not as universal ones. It seems therefore that it is necessary to assign relational properties to the tropes, and in particular: i) define them as existing in spacetime or ii) as belonging to certain bundles. If the tropes, like the particular entities we are accustomed to, are provided with relational properties, the definitional problems related to the persistence of them over time would come back again. If the tropes exist within spacetime, they have relational properties and then we return to the typical difficulties of relational entities in spacetime. Saying that they are defined as belonging to a bundle leads to the same problem because we would be attributing to them relational properties with reference to other tropes. It would be enough the change of one of the tropes in order to change all the tropes to which it is copresent. We might also ask ourselves whether the properties assigned in such a way to tropes are universal immanent. To answer no and say instead that they are tropes themselves means an infinite regress. Such a criticism seems to apply in an invariant manner to any trope theorist who's willing to define each trope apart from each other accordingly to the principle of indiscernibility of identicals. According to such a principle for two entities to be distinct they must differ in at least one property, then what could two red tropes be distinct in if not in their spatial relations?

3 Worm theory

I will distinguish two possible positions: Type x) admits entities of a_1 kind, entities of the $\{a_1, a_2, a_3 \dots a_n\}$ kind (possibly extended to infinity) and entities of the $\{a_1, a_2, a_3, a_6, a_7, a_8\}$ kind. It then admits the set of all instantaneous entities and the power set of it, without the empty set. Type y) admits the set of all stage theory entities and the elements of the power set of it, without the empty set, whose elements are connected, one by one, by the causal relation described by Stage Theory (or by another equivalent). So we will have only entities of the a_1 kind and entities of the $\{a_1, a_2, a_3 \dots a_n\}$ kind.

The way in which physics explains the phenomenon of time dilation provides deliver concrete entities contraddicting Stage theorist's assumptions. Now we are in a different case: both possible positions admit the existence of entities extended over time. One wonders whether the existence of a extended in spacetime, whose identity is guaranteed by Lorentz's invariance for each of its atomic parts, would permit to consider its proper time. If it were like that, it would become absolutely clear in what sense, in the example mentioned above, S_0 , which measures the proper time of a , observes a_5 in t_5 and it would suffice any other persistent entity to define the proper time for S_1 and then say that S_1 observes a_5 at its t_2 . Unfortunately this description includes two distinct problems:

i) As said, with the intent of according with the description of special relativity, I introduced the Lorentz invariance for the definition of the entities. It applies from point to point by determining the length of a vector which establishes the distance between two points in Minkowski spacetime. The points in question are called "events" and the philosophical description immediately notices that these are instantaneous entities $\{a_1, a_2, a_3 \dots\}$. The description of the Worm theory and the relative definition of entities at the changing of their properties is therefore based on the notion of part for these instantaneous entities. It defines each of them and then, by considering them as part of a single entity, it defines the latter. So instantaneous entities $\{a_1, a_2, a_3 \dots\}$ are needed in order to define a . In the aforementioned topic the persistence of a in time is used to define the notion of its own proper time to clarify the sequence of $\{a_1, a_2, a_3 \dots\}$ in special relativity's view but the persistence of a is given only by virtue of the succession of $\{a_1, a_2, a_3 \dots\}$. We are again using the definiendum in the definiens.

ii) As seen before, the type y) requires the cause relation of stage theory or a comparable relation to define entities that it wants existing without accepting those it doesn't want. Again, y) requires instantaneous entities and a relation that binds them in order to define the persistent entities. So it compromises itself with all the difficulties of Stage Theory in the field of time dilation. Type x), on the contrary, has no need for the causal relationship between instantaneous entities to define its entities because, more generously, it includes discontinuous entities persisting in spacetime. The problem is that this very admission makes it conflict with the naive idea of persistent entities of physics. It is not clear how the proper time of an entity of the $\{a_1, a_2, a_8, a_9\}$ kind can be defined. Perhaps it would be possible using entities of the $\{a_1, a_2, a_3 \dots a_9\}$ kind from type y) characterizing them as such with a cause relation or an equivalent. This, however, would involve all the problems related to this type of entity.

By generalizing, we could say that to define entities by Lorentz invariance requires the notion of instantaneous entity and to define the succession of instantaneous entities in Minkowski spacetime requires the notion of proper time. The circularity of definition that I have brought to light above is not at all linked

to the Stage theorist's denial of persistent entities, but to the will to write down physical description in the context of philosophical theories. Again, this does not mean that worm theory cannot agree with the evidence of time dilation. As in stage theory, in fact, we can say that, from distinct reference systems, it is possible to observe distinct instantaneous entities among which physics demands a relation of identity by all means false. Similarly to stage theory, we then have an infinite number of copresent entities independent from us.

4 Considerations

So for what common sense and physics call *a* in stage theory and worm theory relativistic, there are more than infinite entities partially copresent that follow one another overlap with distinct relational properties. Stage Theory does not attribute ontological value to the metrological sum of the entities described as such, while worm theory does. Now, it seems we're bound to introduce some kind of relation that provides a continuity between all those entities. Giving up some kind of relation that provides said continuity would mean to give up any talk in terms of past and future. It would mean to compromise ourselves with the idea that two instants symmetric in time are to be populated by the same objects. This conclusion is not necessarily junk: theories that assume a cyclic time are plenty and they would probably accept that the idea that the world is populated by the same entities each cycle. Even those theories, however, seem to struggle in saying that there's no such relation that guarantees what Sider calls genidentity¹. According to four-dimensionism we perceive identity where it isn't between entities differentiated by their relational properties. In a theory in which any continuity between the entities with relational properties isn't provided, this perception of ours would be unmotivated. It would not be clear why we attribute genidentity to a_1 and a_2 but we don't consider them genidentical to b_1 having ideally the same monadic properties of a_1 . One could think the reason is b_1 shares all its temporal relations with a_1 , but this is not a solution: precisely because a_1 and b_1 differ just for the spatial-relational properties it's not clear how common sense bestows with such certainty genidentity. We may talk about proximity and say that two entities to be genidentical in time must occupy very close regions of spacetime. This position, sadly enough, is subject to a counterexample both for spatial and temporal relations. We perceive genidentity between entities distant in time and space and with teleportation we could soon find ourselves even perceiving genidentity between distant spaces in very short times. A theory that does not account for the great perceptive accord about genidentity seems a stretch but in line with the principle nothing forbids a stretch. On the other hand denying any relation which guarantees such conti-

¹Sider, (1997)

nunity we would also deny the possibility of any predictive assertions. Empirical sciences would be then approximations of an experience which assumes continuity where it isn't. This position seems to be countered by the fact that daily we contemplate the success of our predictive capabilities, based on the pretended continuity of reality. Usually we may say they are based on a continuity which we don't comprehend completely, but here on the contrary we would be negating any continuity and arguing that it's not possible, in principle, to make any predictive assertions. I think that this position is unacceptable and so, from now, I'll assume that some kind of relation that provides continuity between four-dimensional entities must be provided.

In Stage Theory it always was the cause relation that provided continuity. In relativistic perspective, this could imply some difficulties. We can't talk about a set A_1 of infinite entities in t_1 and of a set A_2 of infinite entities in t_2 , this would need an absolute time. Taking into account any instantaneous entity a , it seems difficult to link it with a relation to all entities that are relatively following and that we'd want it to cause, for different reasons. i) we're not legitimated to talk about entities that are relatively following in absence of a relation that provides continuity between the entities and said relation should be the very same cause relation we are trying to define. ii) it's not clear why a should cause $\{a_1, a_2, a_3 \dots\}$ but not b . We're still talking about separate entities with relational and frequently monadic distinct properties. The only one thing that binds a with a_1 and not with b is a continuity we're trying to guarantee precisely with cause relation.

ii) could be solved by saying that each entity causes all entities that are relatively after that. To solve i) we could argue that either the cause relation that allows us to guarantee past and future or some direction of temporal dimension are primitive. I'll consider those position as equivalent and show that in the description we've given there's no clear relation between the entities we're talking about. Let's take three observers $\{s_1, s_2, s_3\}$ that observe each other. If every frame of reference observes separate entities existing independently from the fact that they are observed, then s_1 will see $\{s_1^1, s_2^1, s_3^1\}$ while s_2 will see $\{s_2^2, s_1^2, s_3^2\}$ and s_3 will see $\{s_3^3, s_1^3, s_2^3\}$. Therefore not only relational properties that connect s_1^1 with s_1^2 and s_1^3 aren't clear, but also relations between s_1^1 and, for instance, s_2^2 aren't. How could we say how distant is s_1^1 from s_2^2 ? Apparently no relational property can be posed between such defined entities, included the causal relation.

Apparently we're getting closer to a description in which each of the entities observable from a frame of reference doesn't have any relational properties with entities observable from a different frame of reference. We'd be then in the situation in which to each frame of reference corresponds a universe, the entities of which would be completely unrestrained from any relation with the entities that populate the others. In fact it doesn't seem possible to attribute any clear

relation between the two entities a_1 and a_2 observed from separate frames of reference. In principle this position, no matter how counterintuitive, remains acceptable regarding only the relational properties of spatiotemporal distance. However including also the cause relation, we create a problem: to solve point ii) in fact, we admitted that each entity causes all entities that follow it. Now we're saying that each entity has relations only with the only entities observable from the same frame of reference it can be observed from. Doing so we wouldn't have any cause relation between every stage theory entity, so even introducing cause relation as primitive we wouldn't account for the continuity. Even assuming causation as primitive, it must be a relation only observed or hypothesized which binds the entities in time guaranteeing a continuity between the entities we observe from separate frames of reference. Otherwise we wouldn't have any continuity between the entities. But it doesn't seem possible to attribute any clear relation between entities observable from distinct frames of reference and we have a propensity in negating any relation. That would exclude cause relation and with it the continuity, even as primitive.

Worm theory could think to be free from the need of defining cause relation in order to have continuity between its entities only because it consider metaphysically significant the mereological sum of those entities. Actually this seems to make the problem more difficult, not easier. In fact, not only you have to establish continuity between a_1 and a_2 but also with the temporarily extended entity $\{a_1, a_2\}$, each one existing independently from each other. Having a multiplicity of relational entities in its own spacetime it's restrained by said reasons from guaranteeing a continuity between them and this brings all the stage theory issues.

Note that even in the case I discarded above of entities primitively individuated by monadic properties this objection is available. In fact entities located by primitive monadic properties couldn't be located with respect to Lorentz's invariance, which is a relational property. One may want to say that entities could be individuated by monadic spacetime properties but the problems with proper time don't allows us to do so. In fact just like in stage theory we have a new entity at the variation of relational properties. In this position we have a new entity every time the primitive monadic properties change. Then we have stages just like in stage theory and the same problems with proper time (as shown for worm theory it doesn't matter if we admit or not the mereological sum of those entities). From this we would have different entities partially copresent for every possible frame of reference. So the problems in guaranteeing any relation between entities observed by different frames of reference then problems with the causal relation.

Moreover, I'd like to say that even wondering we can solve the difficulty with proper time, which we need in order to define relational entities on Lorentz in-

variance, a problem will survive. Calosi and Varzi, (2014) solve with multilocation the counterexample from Black to the identity principle for indiscernibles². I will show that multilocation doesn't agree with a position that assume Lorentz invariance as relevant for the definition of his entities, creating problems with the cause relation. Assuming we had defined somehow the notion of proper time, which avoids us the issue of having independent entities for each frame of reference. I'll show that the position which assumes Lorentz invariance in order to have a multiplicity of entities doesn't accord with the principle indispensable to allow for this multiplicity of entities. This because the principle can be saved from the counterexample by Black solely thanks to multilocation.

Until now I considered causation without any specification because the only purpose was to show that between the entities seen by different frame of reference in relativistic stage theory and worm theory there can't be any relation. So it didn't matter what account we take for causation. Now I need a more precise formulation in order to run my argument but I'm not going into the debate about causation. For this reason I'll consider two very general positions about causation to which it should be add much more in order to have a good theory but still are something that every definition of causation have to agree with. A theory that somehow define proper time and consider relevant Lorentz invariance should define cause relation either i) to be connected by a timelike vector whose direction is primitive or ii) primitive which links one by one the events in a sequence guaranteeing the timelike vector's direction. Any position about causation between relational entities existing within spacetime must be at least i) or ii), given that much more should be added for a good definition. In this sense I'm considering causation as a relation which at least must be non symmetric and can go only from relatively past events to relatively future events. I treat this in term of directionality of the vector that links the events. If, as I intent to show, buying multilocation both i) and ii) implies causation in the wrong direction (from relatively future event to relatively past events), then we have a problem in general with causation.

Let's take Black's mind experiment with a slight variation: imagine two events a and b connected by a cause relation that goes from a to b and two events a^1, b^1 symmetric to them (therefore also a^1, b^1 will be connected by a cause relation). To avoid that this kind of example disrupts the principle we must admit that a and a^1 are the same entity multilocated and that the same is true for b and b^1 . So we have to admit that a causes b^1 (and obviously a^1 causes b).

Now: in ii) nothing forces that the vector that connects a and b^1 to be timelike or that the vector timelike that connects them has the right direction. It wouldn't be enough to add a clause for the transitivity of cause relation. In fact it, this would only lead to link a^1 with a^n . It wouldn't link, a^1 with b forbidding the sym-

²Black, (1952)

metry between events connected by timelike vectors.

In the case i) it would be impossible to have a symmetry in which a and b^1 are connected by a timelike vector which has the wrong direction. This because in any case in which a and b^1 are connected by a timelike vector they'll also be connected by an asymmetric cause relation (in particular, placing a^1 and b^1 in the past of a : a^1 causes b^1, a, b whereas a causes only b). Interestingly nothing forbids a symmetry in which they are connected by spacelike vectors.

Note that even a hypothetical case iii) in which maximum uncertainty is allowed regarding cause relation, assuming it as a primitive only observable which links events, would leave it vulnerable to this argument. Nothing forbids, in this description, that cause relation between symmetric events connected by timelike vector to be observed and this case described with multilocation creates problems with cause relation because it forces us to link with it events we wouldn't want to.

This leaves us only with case i). This description, completely incompatible with special relativity, should be written assuming cause relation as "being connected by any vector of primitive direction". Basically each couple of events whose invariant distance is composed by a $\Delta t \neq 0$ would be causally related. To this we must add that in said theory it's possible for an event to cause itself, due to consequences of multilocation. In this last position a counterexample is still available: i) can run in different Minkowski spacetimes consider 1) infinite Minkowski spacetime with two spatial dimensions suppressed, 2) finite but endless (that folds in on itself) Minkowski spacetime with two spatial dimensions suppressed.

Imagine in 1) an infinite series of events with spatial coordinate a such that each event would be preceded and followed by an event which has distance t . Then, imagine a second series of events, each one with spatial coordinate b such that each event has the same temporal coordinate of one of the events of coordinate a . I'm going to show that, unless the identity principle of indiscernibles breaks, all this events are the same one, multilocated. Assuming for simplicity's sake a and b positive such that $a > b$, each of the events of spatial coordinate a is caused by infinite events of coordinate a from which it has distance $\{t; 2t; 3t; 4t \dots\}$ and causes infinite events from which it has distance $\{t; 2t; 3t; 4t \dots\}$. Furthermore, it's caused by infinite events of coordinate b from which it has square distance $\{t^2 - (a - b)^2; (2t)^2 - (a - b)^2; (3t)^2 - (a - b)^2; (4t)^2 - (a - b)^2 \dots\}$ and causes infinite events that have coordinate b from which it has square distance $\{t^2 - (a - b)^2; (2t)^2 - (a - b)^2; (3t)^2 - (a - b)^2; (4t)^2 - (a - b)^2 \dots\}$. Each of the events of spatial coordinate b is caused by infinite events from which it has distance $\{t; 2t; 3t; 4t \dots\}$ and causes infinite events from which it has distance $\{t; 2t; 3t; 4t \dots\}$. Furthermore, it's caused by infinite events from which it has square distance $\{t^2 - (a - b)^2; (2t)^2 - (a - b)^2; (3t)^2 - (a - b)^2; (4t)^2 - (a - b)^2 \dots\}$

and causes infinite events from which it has square distance $\{t^2 - (a-b)^2; (2t)^2 - (a-b)^2; (3t)^2 - (a-b)^2; (4t)^2 - (a-b)^2 \dots\}$. If, as shown, we have to admit that each of those events are the same multilocated, then we are compromising ourselves with cause relation that have the wrong direction. Taking into account three of these events with separate temporal coordinates, we'll have that the former causes the other two and the one in the middle causes the most recent. Given that these three events are the same one multilocated, this implies that the latter causes the one in the middle and the former, and clearly the vector that describes said relation has the wrong direction.

In 2) we have a quite different situation. We won't have infinite events anymore but a finite number of events with coordinate a or b such that each one has distance t from the event that precedes and follows it. Again I'm going to show that, unless the identity principle of indiscernibles breaks, all this events are the same one, multilocated. Each event will have distance $\{t; 2t; 3t; \dots; nt\}$ from $2n$ events and have square distance $\{t^2 - (a-b)^2; (2t)^2 - (a-b)^2; (3t)^2 - (a-b)^2; \dots; (nt)^2 - (a-b)^2\}$ from $2n$ events. Considering any event we then have $4n+1$ events from which it has a distance. This implies, given the position described above, the same difficulties of case 1). If the cause relation is "being separated by vectors of primitive direction", each event it caused by $2n$ events and causes $2n$ events, and it implies, for symmetry, that they are all the same entity and so we have unwanted cause relations.

Using the fact that its spacetime folds in on itself, case 2) could redefine cause relation as "being connected by a timelike vector that doesn't necessarily have to be the shortest possible". Therefore, in the aforementioned case, any event could cause the event that precedes it, in the sense that they are separated by a vector of primitive direction with length $(2n-1)t$. In this description, the direction of time would be saved but at a great loss. In a spacetime that folds in on itself, a relation as the one introduced above, binds each event to itself and to all the other events in spacetime. In this meaning the cause relation would be completely inert due to the definition of the relational entities and so this theory would be exactly the same as a theory which doesn't make use of it. This would lead to issues relative to the continuity needed between relational entities existing within spacetime.

So even imagining we have defined somehow the notion of proper time, issues relative to multilocation survive. Multilocation, on the other hand, is essential to avoid that the identity principle of indiscernibles crumbles to the counterexample by Black. The principle is essential for every theory which assumes a multiplicity of entities. Those issues, to my concern insurmountable, are based on the assumption of multiplicity and because of those I suggest to give up this assumption.

5 Quantum mechanics counterexample to the Identity Principle of Indiscernible

One may note that the argumentation above apply to philosophical theories compatible with special relativity, which is not compatible with quantum mechanics so far. This way could be argued that all those arguments could be simply ignored by theories compatible with quantum mechanics, given that the very nature of the two physical theories make impossible so far to have an ontological position compatible with both. My goal is not only to show that a monist position is compatible with both the theories but also to show that quantum mechanics have even deeper problems with the assumption of a multiplicity of relational entities.

We admit relational properties as relevant for the definition of relational entities in order to define apart two entities with all the monadic properties in common. This because we assume the necessity of defining entities apart in order to have a multiplicity in our ontology. If a counterexample is available to this, if it is possible to have two entities with all the properties, even the relational ones, in common, then we have a problem with the identity principle of indiscernible and so we wouldn't be able to define entities apart. Let's take two bosons a and b , which, having integer spin are not subjected to the Pauli's exclusion principle, so can be in the same orbital having the same spin. Say that a and b are at a reasonable distance one from the other with at least a third asymmetric entity which guarantee they have distinct relational properties. Now assume they move with the same speed measured with the same precision, for the Heisenberg's uncertainty principle we would have the same precision about the position of both. Approaching a with b until they share the orbital we would have two distinct entities with all the properties in common. Note that if we later divide again the couple there is no way in principle to say which is a and which is b (this last fundamental epistemic problem won't be considered here). Note that I'm considering bosons in the example because otherwise the Massimi, (2001)'s answer to Margenau, (1950) would be valid. Moreover note that I'm not claiming that bosons are necessarily indiscernible in every case. I accept Muller and Seevinck, (2009)'s position about weak discernibility in the sense they prove it as contingent. They admit case of indiscernibility for bosons which are precisely the ones I use for the counterexample. My aim is not to show that bosons are indiscernible in general but to show that they are at least in one case, which is enough to break the identity principle of indiscernible.

In this case talk in term of multilocation as resolving the Black's counterexample is inappropriate. We're not in front of one entity which occupies two position, those are two entities which occupy the same position having all the monadic properties in common. In quantum mechanics the "cloud of probabil-

ity” in which the particle is located is its position until something interact with it provoking the collapse of the wave function. We shouldn't consider the particle as having a specific trajectory we don't know but as located, with different probability in every point of the cloud. Then we can have entities which share location but are still not identical, in fact in every moment a and b could interact provoking the collapse of the wave function. If the give bosons have a mass the system would have mass double with respect to each boson, given this it seems hard to deny that we have two entities that share all properties.

One may say that this counterexample could be solved treating quantum mechanics in fictionalist terms. He would assert, in other words, that bosons exist only as useful abstraction we need in order to explain phenomena in the macroscopic world. Then the properties of such abstraction couldn't be a counterexample about concrete entities. I think in this case a strumentalist position is not available. For sure at the beginning particles as bosons has been introduced as a theoretical expedient but now we have different experiments which gave empirical confirmation of the existence of such entities. ATLAS detector in the LHC took billions pictures of elemental particles interaction. We are able to reduce the intensity of a laser until introducing in the Mach-Zender interferometer just one photon but it's impossible in principle to introduce half photon. This seems confirm the quantisation of energy specifically in a experiment that confirm wave-particle dualism. Of course from empirical confirmation don't follow existence but it exclude the possibility of posing those entities as mere abstractions. Who deny the existence of bosons have the burden of proof, those are in the number of thing we should account in our ontological theories. The facts that i) we cant have direct experience of those entities and ii) those entitles can be measured only interacting with something don't seems particularly problematic. A notion of concrete entities which accept only directly perceived entities seems problematic by itself excluding because it excludes Neptune or viruses. About ii) we have to consider that every kind of empirical confirmation is based on interaction between entities, maybe quantum mechanics particles are counterintuitive but are given to our experience just like everything else.

Concluding if the counterexample is valid then a theory compatible with quantum mechanics can't have that two entities sharing all the properties must be the same entity. But this is fundamental in order to define entities apart one to the others, it's fundamental, basically, for every theory assume a multiplicity of relational entities.

Taking again into account a position in which no relational properties are considered and the relevant entities are primitively individuated by monadic properties would bring to the very same difficulty. In this case a and b would still share all the properties, and the identity principle still brake, the only difference is that they don't share anymore position as relational but as monadic

property.

Conclusions

In light of those arguments I suggest to give up the assumption of multiplicity and therefore support the idea that the object of our sensible experience is only one entity. I mean an existence monism in which the only thing charged with ontological value is the whole and our talks about entities are considered as taking into account section of the whole, sectioned by finite set of properties. Obviously, any identity principle of indiscernibles wouldn't be needed, having only one entity loaded with ontological value. The appearance of things would merely be an arbitrary selection of parts of this only object, which allows us to solve both the identity problem and many other philosophically relevant issues. It could seem that sections still need a principle of identity thanks to which we can distinguish a section from another, but because sections don't have any independent existence this necessity falls apart.

Saying "consider two sections a and b such that a has the properties (or, more properly: is selected by criteria) F and Q and b has the properties P and R " the fact of having defined them as two section is everything we need to distinguish them. Nothings forbids us to say "consider two sections such that each has the properties F and Q " and the possibility to distinguish them (in principle, not epistemologically) will be given by the fact that they have been sectioned apart from each other. The fact that two section of the universe so described can share all the properties which have been used to select them, wouldn't be a difficulty. Even a perspective that assumes that a single section could be discernible from itself in given circumstances would be acceptable. In a slightly easier example we'd say "consider a section a such that a has either the properties F and Q or P and R " (more explicitly "consider at time t a section a which in time s_0 has the properties F and Q and in space s_1 has the properties P and R "). In this position no identity principle would be required without the needing of defining several entities apart. The only entity can be sectioned in various ways up to extremely counterintuitive sections. The choice of some theories among others would be determined by criteria with which we normally choose a theory or another. Perspectives theoretically possible but extremely counterintuitive will obviously require huge theoretical benefits to be accepted. Actually it could be the case of quantum physics which admits distinct though indiscernible entities.

It's suitable to highlight that in this theory sections are not made up by the person that introduces the perspective. He's free to define them in regards to number and selection criteria but the continuum he experiences is, at least in part, independent from him. This allows the possibility of having empirical acknowledgement as it's normally described from the moment when a point of

view is set. Having a single concrete entity, what we normally consider as a multitude of entities could be described as an arbitrary selection and the properties we considered till now relevant for the definition of entities would be now criteria we'd use to select. Those wouldn't have any existence independent from us. We'd be free to describe the selections either as instantaneous or as having temporal parts according to the selection criteria we'll decide to use, also based on the position we'll assume about the question on whether spacetime is quantized or not.

We wouldn't end up in the problematic Worm Theory case, in which we need continuity between a_1 , a_2 and also $\{a_1; a_2\}$ because none of these things would exist independently from us, but only as a section we "cut" for theoretical purposes. Continuity would be guaranteed by the fact that we have only one entity. More generally we'll be free from the need of talking in terms of cause relation. In an universe considered as an object extended in four dimension, in the same way the right part of a don't cause its left part, its temporal part a_1 doesn't cause a_2 . From this viewpoint we assist and we're part of an entity extended in spacetime each parts of which has properties. The genidentity we affirm and the predictivity of our laws is explained without using the notion of cause relation which becomes describable in fictionalist terms. Distance relation would be merely relations between sectioned parts without any cause relation, which we need in order to have continuity between a multiplicity of entities. This implies the renounce of a strong conception of past and future. Past and future would derive from the causal relation described from a fictionalist viewpoint in order to explain that universe parts have atemporally the properties which time by time we observe and predict.

The fact that from different frames of reference we observe different properties doesn't seem problematic. In fact, because we are not observing entities with ontological value this wouldn't bring us to the sceptical hypothesis and so we wouldn't be committed with the proliferation of entities typical of ontology with a multiplicity of entities.

This theory is not free of complications. Affirming that we don't have any causal relations but only temporal and spatial parts of a single entity we avoid the aforementioned complications but we switch to a strictly deterministic viewpoint. There wouldn't be any place, from this viewpoint, for free will, because every part of the universe would've already been determined. The romantic idea we have of free will, without it's emotional content, could be described as derived from cause relation defined in a fictionalist terms in order to account for atemporality and aspatiality of the single object we observe. But we must consider the evidence of radioactive isotopes decay, which according to quantum physics is in principle unpredictable (without forgetting it's possible to determine the chances for an atom to decay at any given moment, as it is possible to

determine its half-life) and this seems a counterexample to any possible deterministic viewpoint.

I see at least two ways out from this problem: i) admitting the stochastic-only predictability of some events, still necessary. The monist position in fact, doesn't require the causal relation for its determinism. It's given from the fact that the universe is compounded by parts which have properties and it's atemporal and aspatial. It therefore assumes that each part has "already" the properties we did observe, observe and will observe. So in a certain unpredictable moment where "there was" the section isotope *c* there will be the section atom *d* with a loss of energy and radioactivity. The fact that it is not predictable is in accord with a theory that renounces cause relation. The radioactive decay argument is effective in a determinism which needs perfect cause relation but a position which renounces the cause relation doesn't have any problem. Obviously this theory keeps needing to give up free will. ii) otherwise we could cause relation between parts of the universe as a primitive, only observable, and sometimes completely unintelligible, as in the case of isotopes. This wouldn't have the burden of guaranteeing continuity between a multiplicity of entities since we wouldn't have any multiplicity. It would only need to account for the romantic idea of free will and it would be a relation like the others, with which we select. So, if we desire, we could avoid determinism while keeping the advantages of monism. Personally, I prefer case i) for its greater clarity, because it doesn't leave uncertainty cases in which it's not clear how to apply cause relation and for the fact that it's soberer because it avoids to introduce where I think it's not needed.

One may think that this solution is *ad hoc*. It's true only in a really general notion of *ad hoc* solution. In fact, not only it solve the problems I've been talking about but also counterexamples by quantum physics to the identity principle of indiscernibles and it might work with both relativity and quantum physics. Moreover, it will solve other philosophical questions like the ship of Theseus, the personal identity and the question if the statue of Kant or the piece of marble (or bronze) exists. This theory wouldn't ask positive sciences to work differently. Even if it redefines notions which are fundamental for positive sciences, like the notion of motion and entity, the monist position is completely consistent with positive sciences. In the end, and to my advice mostly, monism solves every difficulty with cause relation by negating this relation in the first place.

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