

ON THE SOUNDNESS OF S5

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Abstract

We prove that the characteristic axiom of modal system S5 under the assumption that the modal operators are logical operators, is sound.

Keywords

Key Terms: Necessity, Possibility, S5.

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The characteristic formula of system S5, the system commonly accepted as the logic of metaphysical modalities is:

$$(S5) \text{Pos } P > \text{Nec Pos } P,$$

where ‘Pos’ stands for ‘Possible’ and ‘Nec’ for ‘Necessary’

We intend to show that if ‘Nec’ and ‘Pos’ are thought of as ‘logical necessity’ and ‘logical possibility’ then the characteristic S5 formula is clearly necessarily true.

We first transpose and get (S5)’s logical equivalent:

$$(S5') \text{Pos Nec } P > \text{Nec } P,$$

For if it is possible that P is necessarily true, then it is possible that P is implied by every sentence, that is by P as well. But no contingent proposition can be implied by its negation e.g., $\text{Nec } (\neg P > P) > \text{Nec } P$. Hence P cannot be contingent. And since P is possible, P cannot be a contradiction. Hence (S5’) and thus (S5).

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