

On Blum's Argument for the Soundness of the S5 Axiom

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Abstract

In a recent paper, Blum argues that the characteristic axiom of S5 is sound when the modal operators are interpreted as logical necessity and logical possibility. In this paper, I show that this interpretation eliminates the modal dimension: logical necessity, whether understood as validity or provability, is a global property of formulas, so the argument collapses to a triviality.

Keywords

Modal Logic, Logical Necessity, S5

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In his recent paper, Alex Blum (2026, “On the Soundness of S5,” *The Reasoner* 20(1), pp. 13–14, <https://doi.org/10.54103/1757-0522/29438>) presents an argument for the soundness of the characteristic axiom of S5,

$$Pos P \rightarrow Nec Pos P,$$

on the assumption that the modal operators express logical necessity and logical possibility. The argument proceeds via a purportedly equivalent formulation,

$$Pos Nec P \rightarrow Nec P,$$



together with the claim that if it is possible that P is necessarily true, then P is necessarily true.

Even if we grant this interpretation, the argument does not establish Blum’s intended result. If “ $Nec P$ ” expresses logical necessity, the natural reading is that P is logically valid (i.e., true in every model). Correspondingly, “ $Pos P$ ” expresses a logical possibility in the sense that P is satisfiable (i.e., true in at least one model). On this interpretation, expressions such as $Pos Nec P$ concern the logical status of formulas (e.g., validity and satisfiability) rather than relations between possible situations.

Logical validity is a global semantic property: P is valid if and only if it is true in every model, equivalently, if there is no logically possible situation in which it is false. From this, it follows that validity cannot hold “possibly but not actually.” If P is valid, it is valid in every model; if it is not valid, then there exists a model in which it fails.

The premise $Pos Nec P$ therefore does not introduce genuine modal possibility. If $Nec P$ expresses logical validity, then either P is valid or it is not; there is no coherent sense in which P could be valid without already being valid. Under this interpretation, $Pos Nec P$ collapses into $Nec P$, and the argument reduces to the trivial schema

$$Nec P \rightarrow Nec P.$$

Blum’s description of

$$Pos Nec P \rightarrow Nec P$$

as a “transposition” of the S5 axiom also requires justification. In standard modal logic, the move from

$$Pos P \rightarrow Nec Pos P$$

to

$$Pos\ Nec\ P \rightarrow Nec\ P$$

depends on modal dualities and substitution principles, not on propositional transposition alone. No such justification is provided.

The difficulty does not disappear if we adopt a proof-theoretic reading. If “*Nec P*” is taken to mean that *P* is provable, provability, like validity, is a global property of formulas rather than a world-relative one. In either case, the expression *Pos Nec P* fails to introduce a genuine modal possibility.

The argument therefore does not establish the soundness of the S5 axiom; it only reflects the strength of the interpretation that Blum adopted.

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